Chapter 3

Modeling and Solving LP Problems in a Spreadsheet

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Spreadsheet Modeling & Decision Analysis, 5e
South-Western, Mason, OH, 2008:45-135
ISBN 978-0-324-65663-3

3.0 Introduction

Chapter 2 discussed how to formulate linear programming (LP) problems and how to solve simple, two-variable LP problems graphically. As you might expect, very few real-world LP problems involve only two decision variables. So, the graphical solution approach is of limited value in solving LP problems. However, the discussion of two-variable problems provides a basis for understanding the issues involved in all LP problems and the general strategies for solving them.

For example, every solvable LP problem has a feasible region, and an optimal solution to the problem can be found at some extreme point of this region (assuming the problem is not unbounded). This is true of all LP problems regardless of the number of decision variables. Although it is fairly easy to graph the feasible region for a two-variable LP problem, it is difficult to visualize or graph the feasible region of an LP problem with three variables because such a graph is three-dimensional. If there are more than three variables, it is virtually impossible to visualize or graph the feasible region for an LP problem because such a graph involves more than three dimensions.

Fortunately, several mathematical techniques exist to solve LP problems involving almost any number of variables without visualizing or graphing their feasible regions. These techniques are now built into spreadsheet packages in a way that makes solving LP problems a fairly simple task. So, using the appropriate computer software, you can solve almost any LP problem easily. The main challenge is ensuring that you formulate the LP problem correctly and communicate this formulation to the computer accurately. This chapter shows you how to do this using spreadsheets.

3.1 Spreadsheet Solvers

Excel, Quattro Pro, and Lotus 1-2-3 all come with built-in spreadsheet optimization tools called **solvers**. Their inclusion in these applications demonstrates the importance of LP (and optimization in general). This book uses Excel to illustrate how spreadsheet solvers can solve optimization problems. However, the same concepts and techniques presented here apply to other spreadsheet packages, although certain details of implementation may differ.

You can also solve optimization problems without using a spreadsheet by using a specialized mathematical programming package. A partial list of these packages includes: LINDO, MPSX, CPLEX, and MathPro. Typically, these packages are used by researchers and businesses interested in solving extremely large problems that do not fit conveniently in a spreadsheet.

The Spreadsheet Solver Company

Frontline Systems, Inc. created the solvers in Microsoft Excel, Lotus 1-2-3, and Corel Quattro Pro. Frontline markets enhanced versions of these spreadsheet solvers that offer greater capacity, faster speed, and several ease-of-use features. You can find out more about Frontline Systems and their products by visiting their Web site at http://www.solver.com.

3.2 Solving LP Problems in a Spreadsheet

We will demonstrate the mechanics of using the Solver in Excel by solving the problem faced by Howie Jones, described in Chapter 2. Recall that Howie owns and operates Blue Ridge Hot Tubs, a company that sells two models of hot tubs: the Aqua-Spa and the Hydro-Lux. Howie purchases prefabricated fiberglass hot tub shells and installs a common water pump and the appropriate amount of tubing into each hot tub. Every Aqua-Spa requires 9 hours of labor and 12 feet of tubing; every Hydro-Lux requires 6 hours of labor and 16 feet of tubing. Demand for these products is such that each Aqua-Spa produced can be sold to generate a profit of $350, and each Hydro-Lux produced can be sold to generate a profit of $300. The company expects to have 200 pumps, 1,566 hours of labor, and 2,880 feet of tubing available during the next production cycle. The problem is to determine the optimal number of Aqua-Spas and Hydro-Luxes to produce to maximize profits.

Chapter 2 developed the following LP formulation for the problem Howie faces. In this model, X_1 represents the number of Aqua-Spas to be produced, and X_2 represents the number of Hydro-Luxes to be produced.

MAX:	$350X_1 + 300X_2$	} profit
Subject to:	$1X_1 + 1X_2 \leq 200$	} pump constraint
	$9X_1 + 6X_2 \leq 1{,}566$	} labor constraint
	$12X_1 + 16X_2 \leq 2{,}880$	} tubing constraint
	$1X_1 \geq 0$	} simple lower bound
	$1X_2 \geq 0$	} simple lower bound

So, how do you solve this problem in a spreadsheet? First, you must implement, or build, this model in the spreadsheet.

3.3 The Steps in Implementing an LP Model in a Spreadsheet

The following four steps summarize what must be done to implement any LP problem in a spreadsheet.

1. **Organize the data for the model on the spreadsheet.** The data for the model consist of the coefficients in the objective function, the various coefficients in the

constraints, and the right-hand-side (RHS) values for the constraints. There is usually more than one way to organize the data for a particular problem on a spreadsheet, but you should keep in mind some general guidelines. First, the goal is to organize the data so their purpose and meaning are as clear as possible. Think of your spreadsheet as a management report that needs to communicate clearly the important factors of the problem being solved. To this end, you should spend some time organizing the data for the problem in your mind's eye—visualizing how the data can be laid out logically—before you start typing values in the spreadsheet. Descriptive labels should be placed in the spreadsheet to clearly identify the various data elements. Often, row and column structures of the data in the model can be used in the spreadsheet to facilitate model implementation. (Note that some or all of the coefficients and values for an LP model might be calculated from other data, often referred to as the primary data. It is best to maintain primary data in the spreadsheet and use appropriate formulas to calculate the coefficients and values that are needed for the LP formulation. Then, if the primary data change, appropriate changes will be made automatically in the coefficients for the LP model.)

2. **Reserve separate cells in the spreadsheet to represent each decision variable in the algebraic model.** Although you can use any empty cells in a spreadsheet to represent the decision variables, it is usually best to arrange the cells representing the decision variables in a way that parallels the structure of the data. This is often helpful in setting up formulas for the objective function and constraints. When possible, it is also a good idea to keep the cells representing decision variables in the same area of the spreadsheet. In addition, you should use descriptive labels to clearly identify the meaning of these cells.
3. **Create a formula in a cell in the spreadsheet that corresponds to the objective function in the algebraic model.** The spreadsheet formula corresponding to the objective function is created by referring to the data cells where the objective function coefficients have been entered (or calculated) and to the corresponding cells representing the decision variables.
4. **For each constraint, create a formula in a separate cell in the spreadsheet that corresponds to the left-hand-side (LHS) of the constraint.** The formula corresponding to the LHS of each constraint is created by referring to the data cells where the coefficients for these constraints have been entered (or calculated) and to the appropriate decision variable cells. Many of the constraint formulas have a similar structure. Thus, when possible, you should create constraint formulas that can be copied to implement other constraint formulas. This not only reduces the effort required to implement a model, but also helps avoid hard-to-detect typing errors.

Although each of the previous steps must be performed to implement an LP model in a spreadsheet, they do not have to be performed in the order indicated. It is usually wise to perform step 1 first, followed by step 2. But the order in which steps 3 and 4 are performed often varies from problem to problem.

Also, it is often wise to use shading, background colors, and/or borders to identify the cells representing decision variables, constraints, and the objective function in a model. This allows the user of a spreadsheet to distinguish more readily between cells representing raw data (that can be changed) and other elements of the model. We have more to say about how to design and implement effective spreadsheet models for LP problems. But first, let's see how to use the previous steps to implement a spreadsheet model using our example problem.

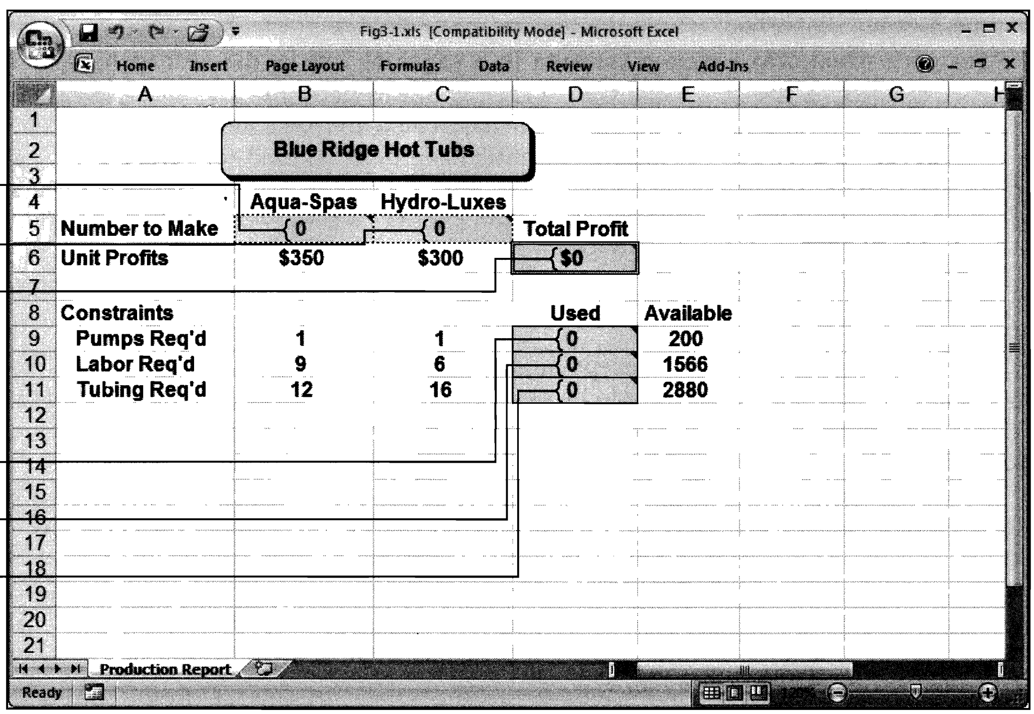
3.4 A Spreadsheet Model for the Blue Ridge Hot Tubs Problem

One possible spreadsheet representation for our example problem is given in Figure 3.1 (and in the file named Fig3-1.xls on your data disk). Let's walk through the creation of this model step-by-step so you can see how it relates to the algebraic formulation of the model.

> **A Note About Macros**
>
> In most of the spreadsheet examples accompanying this book, you can click on the blue title bars at the top of the spreadsheet to toggle on and off a note that provides additional documentation about the spreadsheet model. This documentation feature is enabled through the use of macros. To allow this (and other) macros to run in Excel, click: Office button, Excel options, Trust Center, Trust Center Settings, Macro Settings, select "Disable all macros with notification," click OK. If you then open a file containing macros, Excel displays a security warning indicating some active content has been disabled and will give you the opportunity to enable this content, which you should do to make use of the macro features in the spreadsheet files accompanying this book.

FIGURE 3.1

A spreadsheet model for the Blue Ridge Hot Tub production problem



3.4.1 ORGANIZING THE DATA

One of the first steps in building any spreadsheet model for an LP problem is to organize the data for the model on the spreadsheet. In Figure 3.1, we enter the data for the unit profits for Aqua-Spas and Hydro-Luxes in cells B6 and C6, respectively. Next, we enter the number of pumps, labor hours, and feet of tubing required to produce each type of hot tub, in cells B9 through C11. The values in cells B9 and C9 indicate that one pump is required to produce each type of hot tub. The values in cells B10 and C10 show that each Aqua-Spa produced requires 9 hours of labor, and each Hydro-Lux requires 6 hours. Cells B11 and C11 indicate that each Aqua-Spa produced requires 12 feet of tubing, and each Hydro-Lux requires 16 feet. The available number of pumps, labor hours, and feet of tubing are entered in cells E9 through E11. Notice that appropriate labels also are entered to identify all the data elements for the problem.

3.4.2 REPRESENTING THE DECISION VARIABLES

As indicated in Figure 3.1, cells B5 and C5 represent the decision variables X_1 and X_2 in our algebraic model. These cells are shaded and outlined with dashed borders to distinguish them visually from other elements of the model. Values of zero were placed in cells B5 and C5 because we do not know how many Aqua-Spas and Hydro-Luxes should be produced. Shortly, we will use Solver to determine the optimal values for these cells. Figure 3.2 summarizes the relationship between the decision variables in the algebraic model and the corresponding cells in the spreadsheet.

Decision Variables:	X_1	X_2
	$\downarrow$	$\downarrow$
Spreadsheet Cells:	B5	C5

FIGURE 3.2

Summary of the relationship between the decision variables and corresponding spreadsheet cells

3.4.3 REPRESENTING THE OBJECTIVE FUNCTION

The next step in implementing our LP problem is to create a formula in a cell of the spreadsheet to represent the objective function. We can accomplish this in many ways. Because the objective function is $350X_1 + 300X_2$, you might be tempted to enter the formula =350*B5+300*C5 in the spreadsheet. However, if you wanted to change the coefficients in the objective function, you would have to go back and edit this formula to reflect the changes. Because the objective function coefficients are entered in cells B6 and C6, a better way of implementing the objective function is to refer to the values in cells B6 and C6 rather than entering numeric constants in the formula. The formula for the objective function is entered in cell D6 as:

Formula for cell D6: =B6*B5+C6*C5

As shown in Figure 3.1, cell D6 initially returns the value 0 because cells B5 and C5 both contain zeros. Figure 3.3 summarizes the relationship between the algebraic objective function and the formula entered in cell D6. By implementing the objective function in this manner, if the profits earned on the hot tubs ever change, the spreadsheet model can be changed easily and the problem can be re-solved to determine the effect of this change on the optimal solution. Note that cell D6 has been shaded and outlined with a double border to distinguish it from other elements of the model.

FIGURE 3.3

Summary of the relationship between the decision variables and corresponding spreadsheet cells

Algebraic Objective:	$350\ X_1 + 300\ X_2$
Formula in cell D6:	= B6*B5 + C6*C5

3.4.4 REPRESENTING THE CONSTRAINTS

The next step in building the spreadsheet model involves implementing the constraints of the LP model. Earlier we said that for each constraint in the algebraic model, you must create a formula in a cell of the spreadsheet that corresponds to the LHS of the constraint. The LHS of each constraint in our model is:

LHS of the pump constraint

$$\boxed{1X_1 + 1X_2} \le 200$$

LHS of the labor constraint

$$\boxed{9X_1 + 6X_2} \le 1{,}566$$

LHS of the tubing constraint

$$\boxed{12X_1 + 16X_2} \le 2{,}880$$

We need to set up three cells in the spreadsheet to represent the LHS formulas of the three constraints. Again, we do this by referring to the data cells containing the coefficients for these constraints and to the cells representing the decision variables. The LHS of the first constraint is entered in cell D9 as:

Formula for cell D9: =B9*B5+C9*C5

Similarly, the LHS of the second and third constraints are entered in cells D10 and D11 as:

Formula for cell D10: =B10*B5+C10*C5
Formula for cell D11: =B11*B5+C11*C5

These formulas calculate the number of pumps, hours of labor, and feet of tubing required to manufacture the number of hot tubs represented in cells B5 and C5. Note that cells D9 through D11 were shaded and outlined with solid borders to distinguish them from the other elements of the model.

Figure 3.4 summarizes the relationship between the LHS formulas of the constraints in the algebraic formulation of our model and their spreadsheet representations.

We know that Blue Ridge Hot Tubs has 200 pumps, 1,566 labor hours, and 2,880 feet of tubing available during its next production run. In our algebraic formulation of the LP model, these values represent the RHS values for the three constraints. Therefore, we entered the available number of pumps, hours of labor, and feet of tubing in cells E9, E10, and E11, respectively. These terms indicate the upper limits on the values that cells D9, D10, and D11 can assume.

3.4.5 REPRESENTING THE BOUNDS ON THE DECISION VARIABLES

Now, what about the simple lower bounds on our decision variables represented by $X_1 \ge 0$ and $X_2 \ge 0$? These conditions are quite common in LP problems and are referred

LHS formula for the pump constraint:	$1\ X_1 + 1\ X_2$
Formula in cell D9:	= B9 * B5 + C9 * C5
LHS formula for the labor constraint:	$9\ X_1 + 6\ X_2$
Formula in cell D10:	= B10*B5 + C10*C5
LHS formula for the tubing constraint:	$12\ X_1 + 16\ X_2$
Formula in cell D11:	= B11*B5 + C11*C5

FIGURE 3.4

Summary of the relationship between the LHS formulas of the constraints and their spreadsheet representations

to as **nonnegativity conditions** because they indicate that the decision variables can assume only nonnegative values. These conditions might seem like constraints and can, in fact, be implemented like the other constraints. However, Solver allows you to specify simple upper and lower bounds for the decision variables by referring directly to the cells representing the decision variables. Thus, at this point, we have taken no specific action to implement these bounds in our spreadsheet.

3.5 How Solver Views the Model

After implementing our model in the spreadsheet, we can use Solver to find the optimal solution to the problem. But first, we need to define the following three components of our spreadsheet model for Solver:

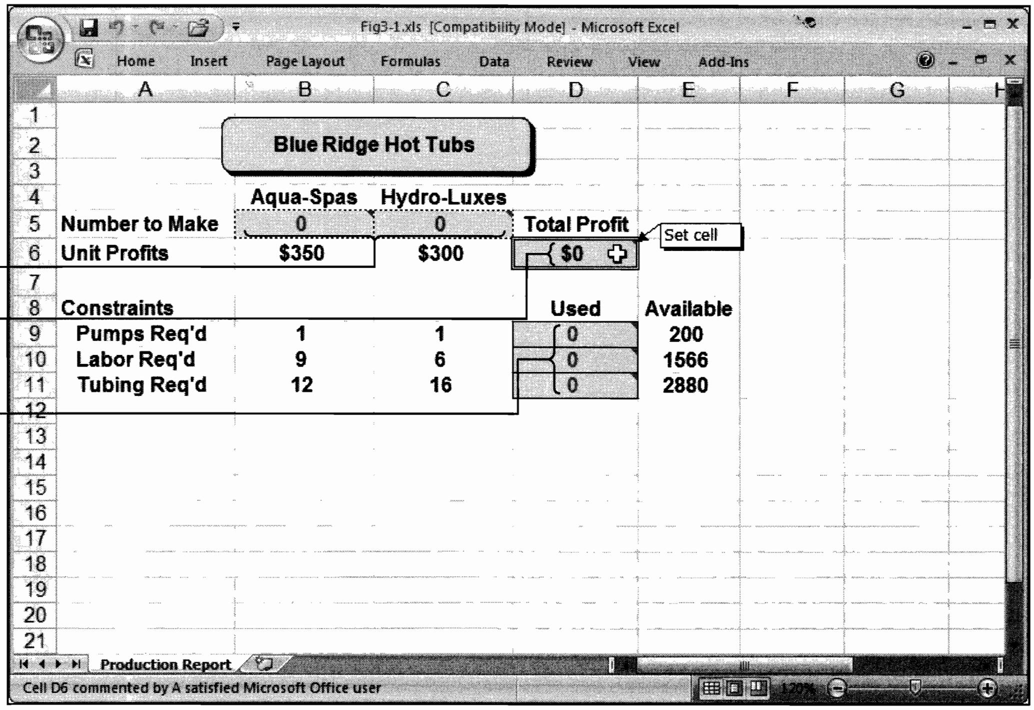
1. **Set (or Target) cell.** The cell in the spreadsheet that represents the *objective function* in the model (and whether its value should be maximized or minimized).
2. **Variable (or Changing) cells.** The cells in the spreadsheet that represent the *decision variables* in the model.
3. **Constraint cells.** The cells in the spreadsheet that represent the *LHS formulas* of the constraints in the model (and any upper and lower bounds that apply to these formulas).

These components correspond directly to the cells in the spreadsheet that we established when implementing the LP model. For example, in the spreadsheet for our example problem, the set (or target) cell is represented by cell D6, the variable (or changing) cells are represented by cells B5 and C5, and the constraint cells are represented by cells D9, D10, and D11. Figure 3.5 shows these relationships. Figure 3.5 also shows a cell note documenting the purpose of cell D6. Cell notes can be a very effective way of describing details about the purpose or meaning of various cells in a model.

By comparing Figure 3.1 with Figure 3.5, you can see the direct connection between the way we formulate LP models algebraically and how Solver views the spreadsheet implementation of the model. The decision variables in the algebraic model correspond to the variable (or changing) cells for Solver. The LHS formulas for the different

FIGURE 3.5

Summary of Solver's view of the model



constraints in the algebraic model correspond to the constraint cells for Solver. Finally, the objective function in the algebraic model corresponds to the set (or target) cell for Solver. So, although the terminology Solver uses to describe spreadsheet LP models is somewhat different from the terminology we use to describe LP models algebraically, the concepts are the same. Figure 3.6 summarizes these differences in terminology.

Note that some versions of Solver refer to the cells containing the objective function as the "target" cell, whereas other versions of Solver refer to it simply as the "set" cell. Similarly, some versions of Solver refer to the cells representing the decision variables as "changing" cells, whereas other versions refer to them as "variable" cells. As a result, we may use the terms "target" cell and "set" cell interchangeably in this book to refer to the cell containing the objective function. Similarly, we may use the terms "changing" cells and "variable" cells interchangeably to refer to cells representing decision variables.

FIGURE 3.6

Summary of Solver terminology

Terms used to describe LP models algebraically	Corresponding terms used by solver to describe spreadsheet LP models
objective function	set (or target) cell
decision variables	variable (or changing) cells
LHS formulas of constraints	constraint cells

A Note About Creating Cell Comments...

It is easy to create cell comments like the one shown for cell D6 in Figure 3.5. To create a comment for a cell:

1. Click the cell to select it.
2. Choose Review, New Comment (or press the Shift key and function key F2 simultaneously).
3. Type the comment for the cell, and then select another cell.

The display of cell comments can be turned on or off as follows:

1. Choose Review.
2. Select the appropriate option in the Comments section.
3. Click the OK button.

To copy a cell comment from one cell to a series of other cells:

1. Click the cell containing the comment you want to copy.
2. Choose the Copy command on the Home, Clipboard ribbon (or press the Ctrl and C keys simultaneously).
3. Select the cells you want to copy the comment to.
4. Select the Paste Special command on the Home, Clipboard, Paste ribbon (or click the right mouse button and select Paste Special).
5. Select the Comments option button.
6. Click the OK button.

Installing Premium Solver for Education

This book comes with Premium Solver for Education—an upgraded version of the standard Solver that ships with Excel. If you have not already done so, install Premium Solver for Education now by running the program called PremSolv.exe found on the CD-ROM that accompanies this book. To do this, use Windows Explorer to locate the file named PremSolv.exe and then double-click the file name. (If you are running Excel in a networked environment, consult with your network administrator.) Although most of the examples in this book also work with the standard Solver that comes with Excel, Premium Solver for Education includes several helpful features that are discussed throughout this book.

3.6 Using Solver

After implementing an LP model in a spreadsheet, we still need to solve the model. To do this, we must first indicate to Solver which cells in the spreadsheet represent the objective function, the decision variables, and the constraints. To invoke Solver in Excel,

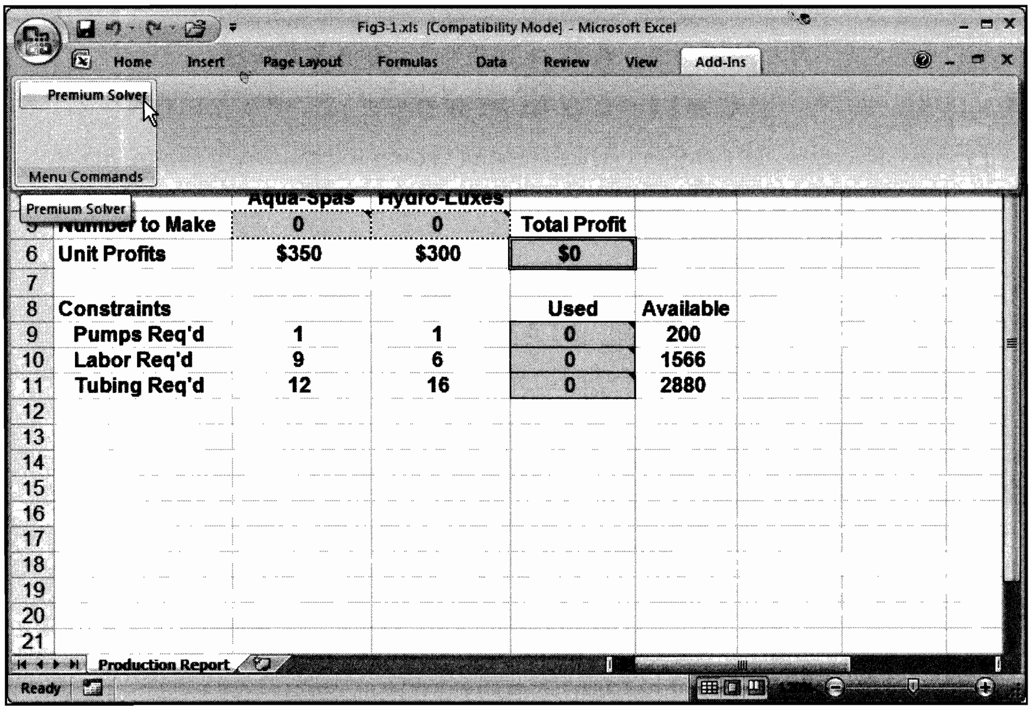


FIGURE 3.7

Command for invoking Solver

choose the Solver command from the Add-Ins menu, as shown in Figure 3.7. This should display the first Solver Parameters dialog box shown in Figure 3.8.

A different "standard" version of Solver ships with Excel and is normally found under Excel's Data, Analysis command. The interface of the standard Solver is the second dialog shown in Figure 3.8. Premium Solver is more powerful than the standard Solver and, as a result, we will be using it throughout this book. However, you might sometimes encounter standard Solver on different computers.

Premium Solver for Education provides three different algorithms for solving optimization problems: Standard GRG Nonlinear, Standard Simplex LP, and Standard Evolutionary. If the problem you are trying to solve is an LP problem (that is, an optimization problem with a linear objective function and linear constraints), Solver can use a special algorithm known as the ***simplex method*** to solve the problem. The simplex method provides an efficient way of solving LP problems and, therefore, requires less solution time. Furthermore, using the simplex method allows for expanded sensitivity information about the solution obtained. (Chapter 4 discusses this in detail.) In any event, when using Solver to solve an LP problem, it is a good idea to select the Standard Simplex LP option, as indicated in Figure 3.8.

3.6.1 DEFINING THE SET (OR TARGET) CELL

In the Solver Parameters dialog box, specify the location of the cell that represents the objective function by entering it in the Set Cell box, as shown in Figure 3.9.

Notice that cell D6 contains a formula representing the objective function for our problem and that we instructed Solver to try to maximize this value, as specified by the Max button. Select the Min button when you want Solver to find a solution that minimizes the value of the objective. The Value button may be used to find a solution for which the objective function takes on a specific value.

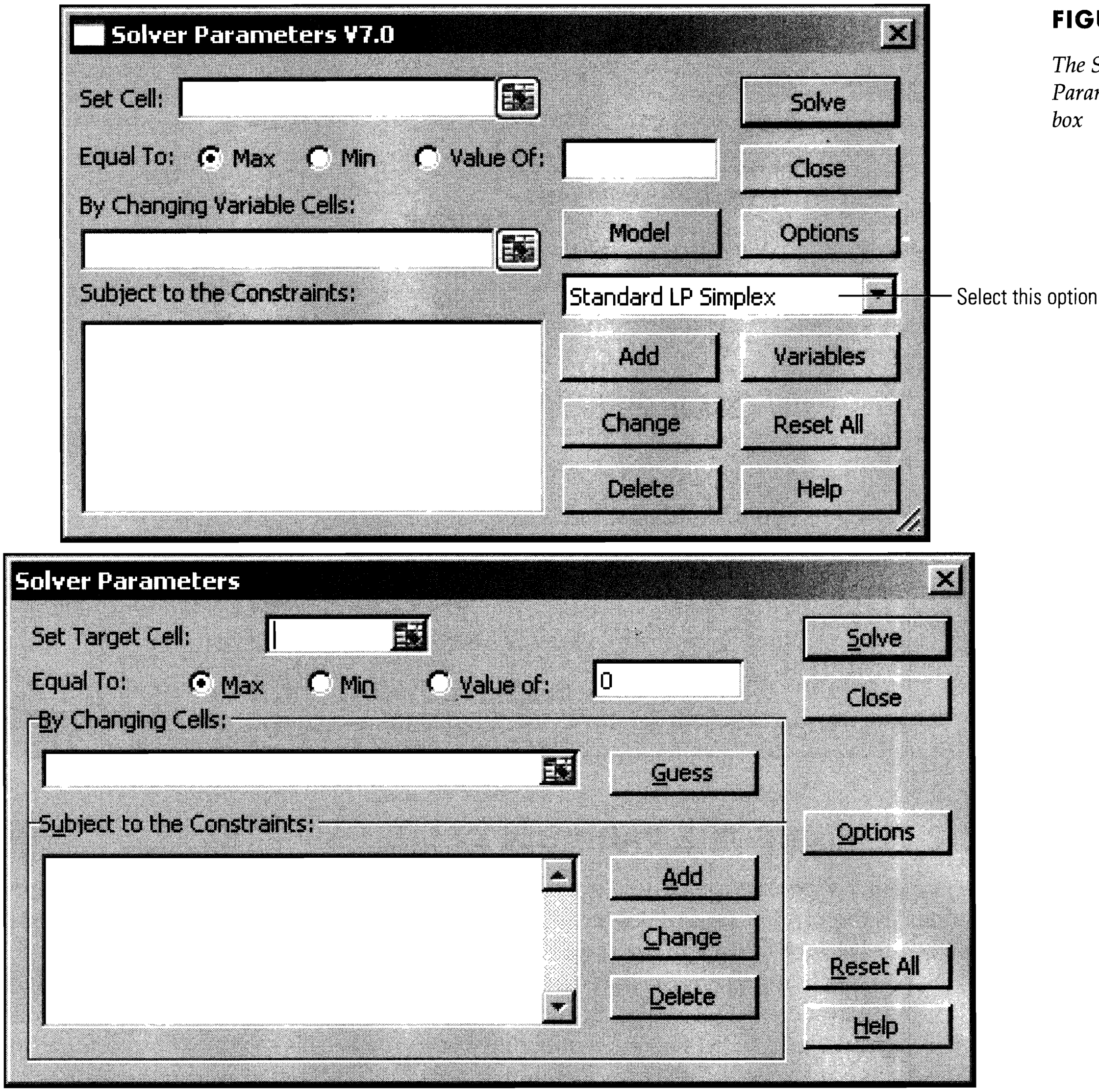


FIGURE 3.8

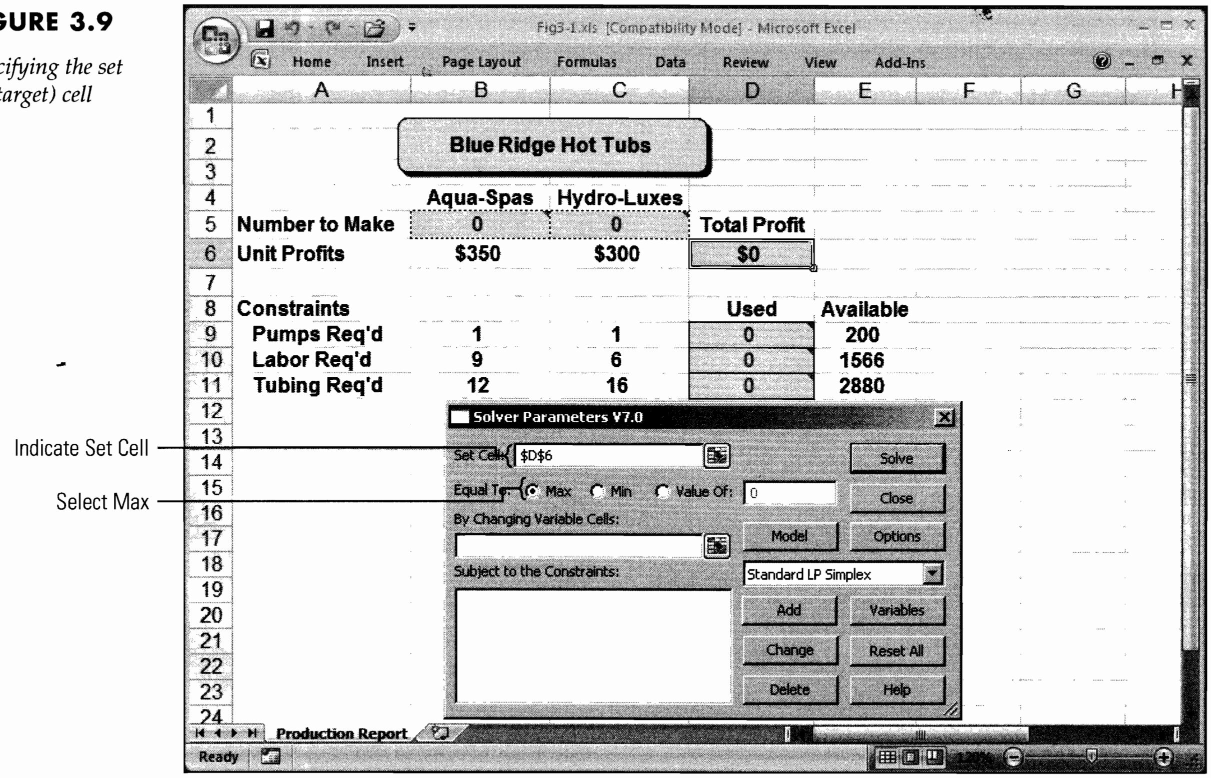
The Solver Parameters dialog box

An Evolutionary Force in Spreadsheets

Dan Fylstra is the president of Frontline Systems, the company that created Solver. He was also one of the instrumental people behind the release of the first spreadsheet program, VisiCalc. Dan received his bachelor's degree at MIT in 1975 and went on to get his master's of business from Harvard Business School in 1978. A Harvard professor introduced Fylstra to another Harvard student named Dan Bricklin, who had an idea for a software program that would let users enter numbers in an electronic spreadsheet, automatically calculating the results on-screen. Bricklin was the idea man and his friend, Bob Frankston, would write the program. Fylstra loaned Bricklin the Apple computer to write the program in 1978. Source: www.smartcomputing.com, *May 2002 • Vol. 6 Issue 5*

FIGURE 3.9

Specifying the set (or target) cell



3.6.2 DEFINING THE VARIABLE CELLS

To solve our LP problem, we also need to indicate which cells represent the decision variables in the model. Again, Solver refers to these cells as variable cells. The variable cells for our example problem are specified as shown in Figure 3.10.

Cells B5 and C5 represent the decision variables for the model. Solver will determine the optimal values for these cells. If the decision variables were not in a contiguous range, we would have to list the individual decision variable cells separated by commas in the By Changing Variable Cells box. Whenever possible, it is best to use contiguous cells to represent the decision variables.

3.6.3 DEFINING THE CONSTRAINT CELLS

Next, we must define the constraint cells in the spreadsheet and the restrictions that apply to these cells. As mentioned earlier, the constraint cells are the cells in which we implemented the LHS formulas for each constraint in our model. To define the constraint cells, click the Add button shown in Figure 3.10, and then complete the Add Constraint dialog box shown in Figure 3.11. In the Add Constraint dialog box, click the Add button again to define additional constraints. Click the OK button when you have finished defining constraints.

Cells D9 through D11 represent constraint cells whose values must be less than or equal to the values in cells E9 through E11, respectively. If the constraint cells were not in contiguous cells in the spreadsheet, we would have to define the constraint cells repeatedly. As with the variable cells, it usually is best to choose contiguous cells in your spreadsheet to implement the LHS formulas of the constraints in a model.

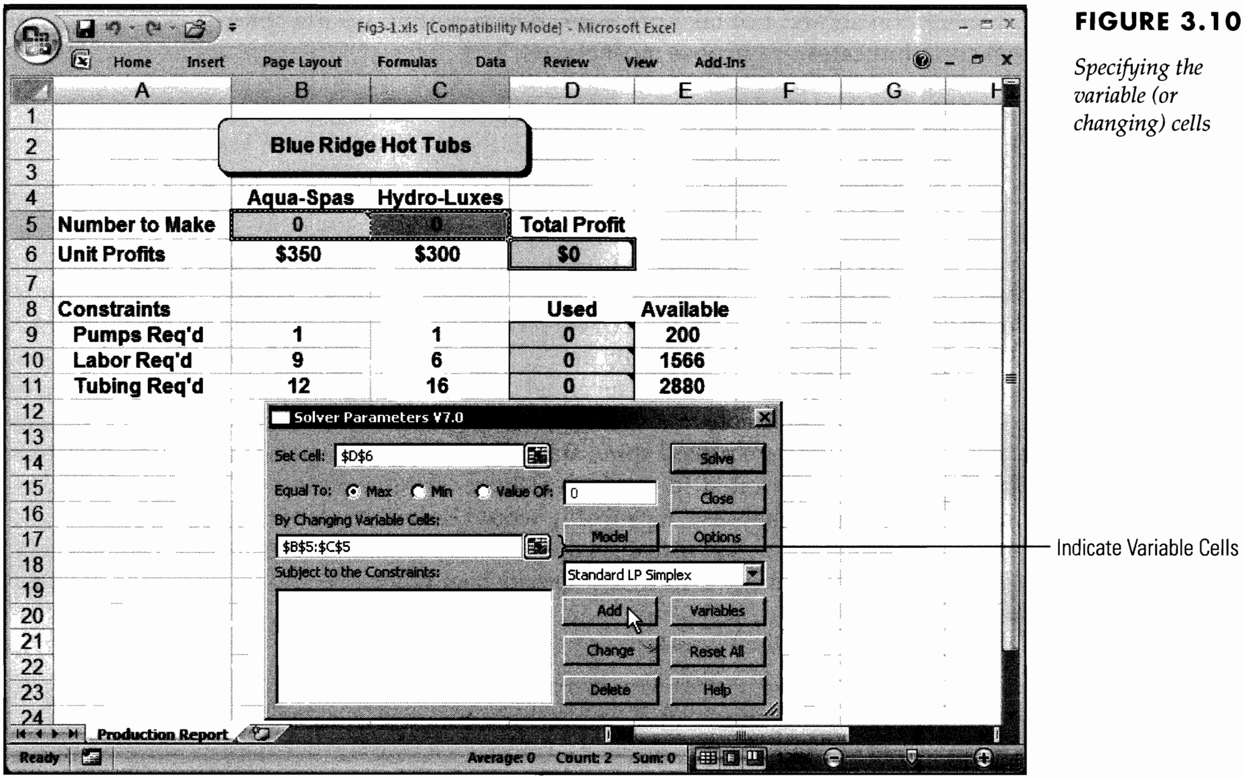


FIGURE 3.10

Specifying the variable (or changing) cells

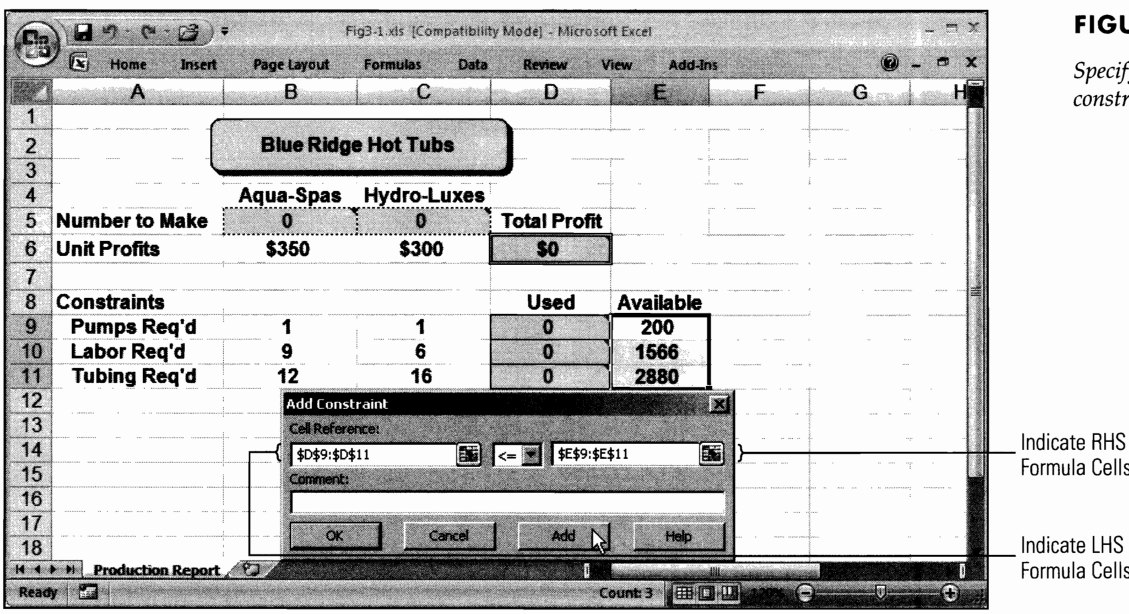


FIGURE 3.11

Specifying the constraint cells

If you want to define more than one constraint at the same time, as in Figure 3.11, all the constraint cells you select must be the same type (that is, they must all be $\leq$, $\geq$, or $=$). Therefore, it is a good idea to keep constraints of a given type grouped in contiguous cells so that you can select them at the same time. For example, in our case, the three constraint cells we selected are all "less than or equal to" ($\leq$) constraints. However, this consideration should not take precedence over setting up the spreadsheet in the way that communicates its purpose most clearly.

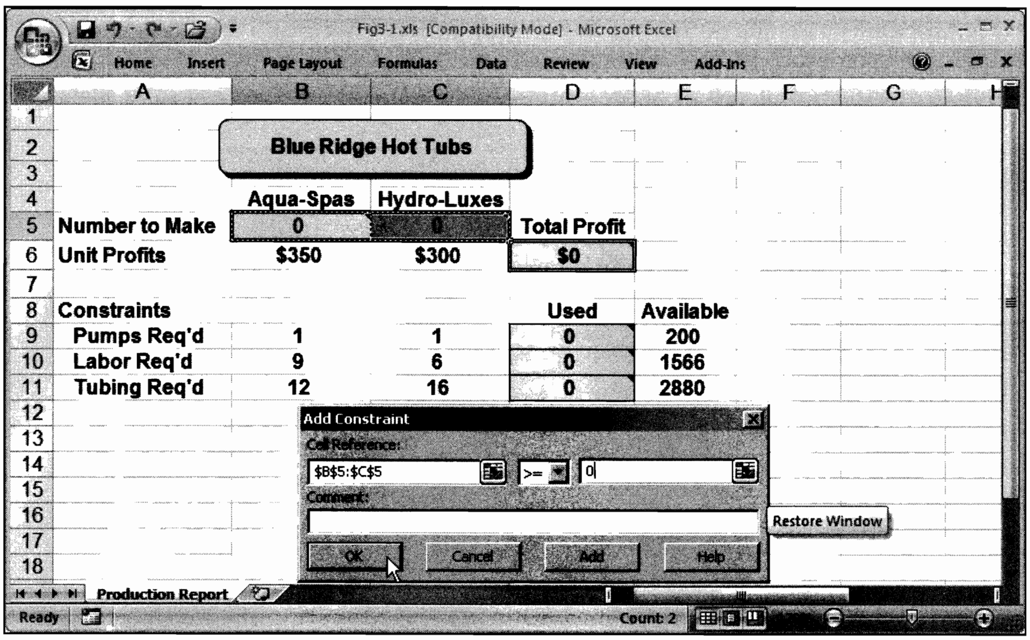
3.6.4 DEFINING THE NONNEGATIVITY CONDITIONS

One final specification we need to make for our model is that the decision variables must be greater than or equal to zero. As mentioned earlier, we can impose these conditions as constraints by placing appropriate restrictions on the values that can be assigned to the cells representing the decision variables (in this case, cells B5 and C5). To do this, we simply add another set of constraints to the model, as shown in Figure 3.12.

Figure 3.12 indicates that cells B5 and C5, which represent the decision variables in our model, must be greater than or equal to zero. Notice that the RHS value of this constraint is a numeric constant that is entered manually. The same type of constraints also could be used if we placed some strictly positive lower bounds on these variables (for example, if we wanted to produce at least 10 Aqua-Spas and at least 10 Hydro-Luxes). However, in that case, it probably would be best to place the minimum required production amounts on the spreadsheet so that these restrictions are displayed clearly. We can then refer to those cells in the spreadsheet when specifying the RHS values for these constraints.

FIGURE 3.12

Adding the nonnegativity conditions for the problem



Important Software Note

There is another way to impose nonnegativity conditions—just check the Assume Non-Negative check box in the Solver Options dialog box (shown in Figure 3.14.) Checking this box tells Solver to assume that all the variables (or variable cells) in your model that have not been assigned explicit lower bounds should have lower bounds of zero.

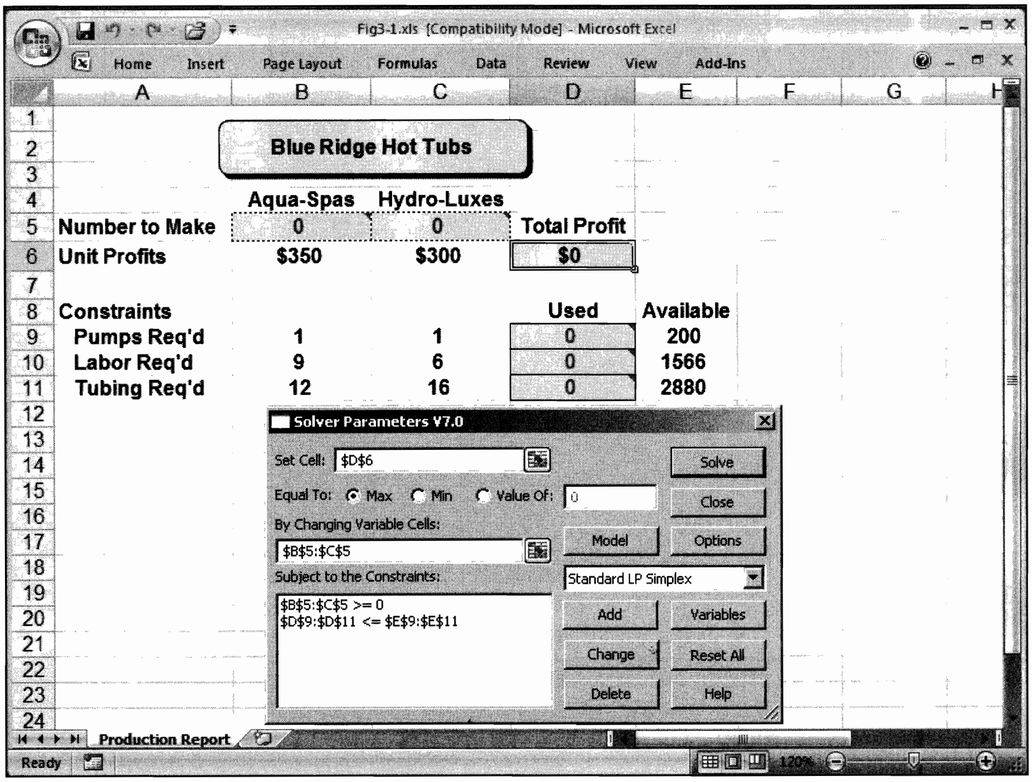


FIGURE 3.13

Summary of how Solver views the model

3.6.5 REVIEWING THE MODEL

After specifying all the constraints for our model, the final Solver Parameters dialog box appears, as shown in Figure 3.13. This dialog box provides a summary of how Solver views our model. It is a good idea to review this information before solving the model to make sure that you entered all the parameters accurately, and to correct any errors before proceeding.

3.6.6 OPTIONS

Solver provides several options that affect how it solves a problem. These options are available in the Solver Options dialog box, which you display by clicking the Options button in the Solver Parameters dialog box. Figure 3.14 shows the Solver Options dialog box for the Standard Simplex LP solution algorithm. We will discuss the meanings of several of these options as we proceed. You also can find out more about these options by clicking the Help button in the Solver Options dialog box.

3.6.7 SOLVING THE MODEL

After entering all the appropriate parameters and choosing any necessary options for our model, the next step is to solve the problem. Click the Solve button in the Solver Parameters dialog box to solve the problem. When Solver finds the optimal solution, it displays the Solver Results dialog box shown in Figure 3.15. If the values on your screen do not match those in Figure 3.15, click the Restore Original Values option button, click OK, and try again.

This dialog box provides options for keeping the solution found by Solver or restoring the spreadsheet to its original condition. Most often, you will want to keep Solver's solution unless there is an obvious problem with it. Notice that the Solver Results dialog

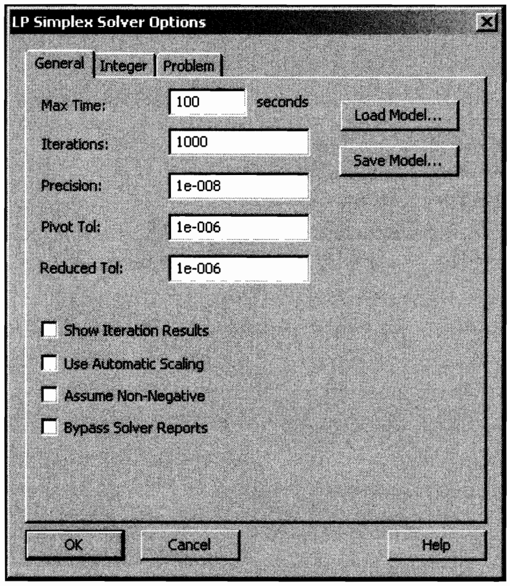


FIGURE 3.14

The Solver Options dialog box

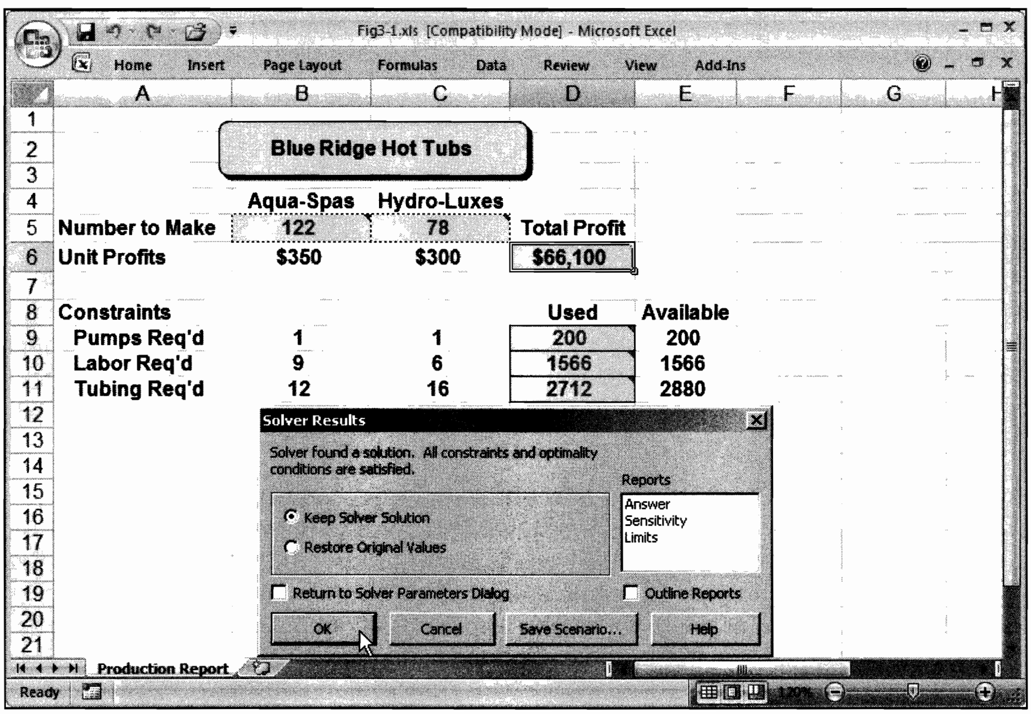


FIGURE 3.15

Optimal solution for the Blue Ridge Hot Tubs problem

box also provides options for generating Answer, Sensitivity, and Limits reports. Chapter 4 discusses these options.

As shown in Figure 3.15, Solver determined that the optimal value for cell B5 is 122 and the optimal value for cell C5 is 78. These values correspond to the optimal values for X_1 and X_2 that we determined graphically in Chapter 2. The value of the set cell (D6) now indicates that if Blue Ridge Hot Tubs produces and sells 122 Aqua-Spas and 78 Hydro-Luxes, the company will earn a profit of $66,100. Cells D9, D10, and D11 indicate that this solution uses all the 200 available pumps, all the 1,566 available labor hours, and 2,712 of the 2,880 feet of available tubing.

3.7 Goals and Guidelines for Spreadsheet Design

Now that you have a basic idea of how Solver works and how to set up an LP model in a spreadsheet, we'll walk through several more examples of formulating LP models and solving them with Solver. These problems highlight the wide variety of business problems in which LP can be applied and also will show you some helpful "tricks of the trade" that should help you solve the problems at the end of this chapter. When you work through the end-of-the-chapter problems, you will better appreciate how much thought is required to find a good way to implement a given model.

As we proceed, keep in mind that you can set up these problems more than one way. Creating spreadsheet models that communicate their purpose effectively is very much an art—or at least an acquired skill. Spreadsheets are inherently free-form and impose no particular structure on the way we model problems. As a result, there is no one "right" way to model a problem in a spreadsheet; however some ways certainly are better (or more logical) than others. To achieve the end result of a logical spreadsheet design, your modeling efforts should be directed toward the following goals:

- **Communication.** A spreadsheet's primary business purpose is that of communicating information to managers. As such, the primary design objective in most spreadsheet modeling tasks is to communicate the relevant aspects of the problem at hand in as clear and intuitively appealing a manner as possible.
- **Reliability.** The output that a spreadsheet generates should be correct and consistent. This has an obvious impact on the degree of confidence a manager places in the results of the modeling effort.
- **Auditability.** A manager should be able to retrace the steps followed to generate the different outputs from the model, to understand the model and to verify results. Models that are set up in an intuitively appealing, logical layout tend to be the most auditable.
- **Modifiability.** The data and assumptions upon which we build spreadsheet models can change frequently. A well-designed spreadsheet should be easy to change or enhance to meet changing user requirements.

In most cases, the spreadsheet design that communicates its purpose most clearly also will be the most reliable, auditable, and modifiable design. As you consider different ways of implementing a spreadsheet model for a particular problem, consider how well the modeling alternatives compare in terms of these goals. Some practical suggestions and guidelines for creating effective spreadsheet models are given in Figure 3.16.

FIGURE 3.16

Guidelines for effective spreadsheet design

Spreadsheet Design Guidelines

- **Organize the data, then build the model around the data.** After the data is arranged in a visually appealing manner, logical locations for decision variables, constraints, and the objective function tend to naturally suggest themselves. This also tends to enhance the reliability, auditability, and maintainability of the model.
- **Do not embed numeric constants in formulas.** Numeric constants should be placed in individual cells and labeled appropriately. This enhances the reliability and modifiability of the model.
- **Things which are logically related (for example, LHS and RHS of constraints) should be arranged in close physical proximity to one another and in the same columnar or row orientation.** This enhances reliability and auditability of the model.

(Continued)

- **A design that results in formulas that can be copied is probably better than one that does not.** A model with formulas that can copied to complete a series of calculations in a range is less prone to error (more reliable) and tends to be more understandable (auditable). Once users understand the first formula in a range, they understand all the formulas in a range.
- **Column or row totals should be in close proximity to the columns or rows being totaled.** Spreadsheet users often expect numbers at the end of a column or row to represent a total or some other summary measure involving the data in the column or row. Numbers at the ends of columns or rows that do not represent totals can be misinterpreted easily (reducing auditability).
- **The English-reading human eye scans left to right, top to bottom.** This fact should be considered and reflected in the spreadsheet design to enhance the auditability of the model.
- **Use color, shading, borders, and protection to distinguish changeable parameters from other elements of the model.** This enhances the reliability and modifiability of the model.
- **Use text boxes and cell comments to document various elements of the model.** These devices can be used to provide greater detail about a model or particular cells in a model than labels on a spreadsheet might allow.

Spreadsheet-Based LP Solvers Create New Applications for Linear Programming

In 1987, *The Wall Street Journal* reported on an exciting new trend in business—the availability of solvers for personal computers that allowed many businesses to transfer LP models from mainframe computers. Newfoundland Energy Ltd., for example, had evaluated its mix of crude oils to purchase with LP on a mainframe for 25 years. Since it began using a personal computer for this application, the company has saved thousands of dollars per year in mainframe access time charges.

The expansion of access to LP also spawned new applications. Therese Fitzpatrick, a nursing administrator at Grant Hospital in Chicago, used spreadsheet optimization to create a staff scheduling model that was projected to save the hospital $80,000 per month in overtime and temporary hiring costs. The task of scheduling 300 nurses so that those with appropriate skills were in the right place at the right time required 20 hours per month. The LP model enabled Therese to do the job in four hours, even with such complicating factors as leaves, vacations, and variations in staffing requirements at different times and days of the week.

Hawley Fuel Corp., a New York wholesaler of coal, found that it could minimize its cost of purchases while still meeting customers' requirements for sulfur and ash content by optimizing a spreadsheet LP model. Charles Howard of Victoria, British Columbia, developed an LP model to increase electricity generation from a dam just by opening and closing the outlet valves at the right time.

(Source: Bulkely, William M., "The Right Mix: New Software Makes the Choice Much Easier," *The Wall Street Journal*, March 27, 1987, p. 17.)

3.8 Make vs. Buy Decisions

As mentioned at the beginning of Chapter 2, LP is particularly well-suited to problems where scarce or limited resources must be allocated or used in an optimal manner. Numerous examples of these types of problems occur in manufacturing organizations. For example, LP might be used to determine how the various components of a job should be assigned to multipurpose machines to minimize the time it takes to complete the job. As another example, a company might receive an order for several items that it cannot fill entirely with its own production capacity. In such a case, the company must determine which items to produce and which items to subcontract (or buy) from an outside supplier. The following is an example of this type of make vs. buy decision.

> The Electro-Poly Corporation is the world's leading manufacturer of slip rings. A slip ring is an electrical coupling device that allows current to pass through a spinning or rotating connection—such as a gun turret on a ship, aircraft, or tank. The company recently received a $750,000 order for various quantities of three types of slip rings. Each slip ring requires a certain amount of time to wire and harness. The following table summarizes the requirements for the three models of slip rings.
>
> | | Model 1 | Model 2 | Model 3 |
> |---|---|---|---|
> | Number Ordered | 3,000 | 2,000 | 900 |
> | Hours of Wiring Required per Unit | 2 | 1.5 | 3 |
> | Hours of Harnessing Required per Unit | 1 | 2 | 1 |
>
> Unfortunately, Electro-Poly does not have enough wiring and harnessing capacity to fill the order by its due date. The company has only 10,000 hours of wiring capacity and 5,000 hours of harnessing capacity available to devote to this order. However, the company can subcontract any portion of this order to one of its competitors. The unit costs of producing each model in-house and buying the finished products from a competitor are summarized below.
>
> | | Model 1 | Model 2 | Model 3 |
> |---|---|---|---|
> | Cost to Make | $50 | $83 | $130 |
> | Cost to Buy | $61 | $97 | $145 |
>
> Electro-Poly wants to determine the number of slip rings to make and the number to buy to fill the customer order at the least possible cost.

3.8.1 DEFINING THE DECISION VARIABLES

To solve the Electro-Poly problem, we need six decision variables to represent the alternatives under consideration. The six variables are:

M_1 = number of model 1 slip rings to make in-house
M_2 = number of model 2 slip rings to make in-house
M_3 = number of model 3 slip rings to make in-house
B_1 = number of model 1 slip rings to buy from competitor
B_2 = number of model 2 slip rings to buy from competitor
B_3 = number of model 3 slip rings to buy from competitor

As mentioned in Chapter 2, we do not have to use the symbols $X_1, X_2, \ldots, X_n$ for the decision variables. If other symbols better clarify the model, you are certainly free to use them. In this case, the symbols M_i and B_i help distinguish the **M**ake in-house variables from the **B**uy from competitor variables.

3.8.2 DEFINING THE OBJECTIVE FUNCTION

The objective in this problem is to minimize the total cost of filling the order. Recall that each model 1 slip ring made in-house (each unit of M_1) costs \$50; each model 2 slip ring made in-house (each unit of M_2) costs \$83; and each model 3 slip ring (each unit of M_3) costs \$130. Each model 1 slip ring bought from the competitor (each unit of B_1) costs \$61; each model 2 slip ring bought from the competitor (each unit of B_2) costs \$97; and each model 3 slip ring bought from the competitor (each unit of B_3) costs \$145. Thus, the objective is stated mathematically as:

$$\text{MIN:} \quad 50M_1 + 83M_2 + 130M_3 + 61B_1 + 97B_2 + 145B_3$$

3.8.3 DEFINING THE CONSTRAINTS

Several constraints affect this problem. Two constraints are needed to ensure that the number of slip rings made in-house does not exceed the available capacity for wiring and harnessing. These constraints are stated as:

$$\begin{array}{lllllll} 2M_1 & + & 1.5M_2 & + & 3M_3 & \leq & 10{,}000 \quad \} \text{ wiring constraint} \\ 1M_1 & + & 2M_2 & + & 1M_3 & \leq & 5{,}000 \quad \} \text{ harnessing constraint} \end{array}$$

Three additional constraints ensure that 3,000 model 1 slip rings, 2,000 model 2 slip rings, and 900 model 3 slip rings are available to fill the order. These constraints are stated as:

$$\begin{array}{lllll} M_1 & + & B_1 & = & 3{,}000 \quad \} \text{ demand for model 1} \\ M_2 & + & B_2 & = & 2{,}000 \quad \} \text{ demand for model 2} \\ M_3 & + & B_3 & = & 900 \quad \} \text{ demand for model 3} \end{array}$$

Finally, because none of the variables in the model can assume a value of less than zero, we also need the following nonnegativity condition:

$$M_1, M_2, M_3, B_1, B_2, B_3 \geq 0$$

3.8.4 IMPLEMENTING THE MODEL

The LP model for Electro-Poly's make vs. buy problem is summarized as:

$$\begin{array}{llllllll} \text{MIN:} & \multicolumn{5}{l}{50M_1 + 83M_2 + 130M_3 + 61B_1 + 97B_2 + 145B_3} & & \} \text{ total cost} \\ \text{Subject to:} & M_1 & + & B_1 & & = & 3{,}000 & \} \text{ demand for model 1} \\ & M_2 & + & B_2 & & = & 2{,}000 & \} \text{ demand for model 2} \\ & M_3 & + & B_3 & & = & 900 & \} \text{ demand for model 3} \\ & 2M_1 & + & 1.5M_2 & + \; 3M_3 & \leq & 10{,}000 & \} \text{ wiring constraint} \\ & 1M_1 & + & 2M_2 & + \; 1M_3 & \leq & 5{,}000 & \} \text{ harnessing constraint} \\ & & & \multicolumn{2}{l}{M_1, M_2, M_3, B_1, B_2, B_3} & \geq & 0 & \} \text{ nonnegativity condition} \end{array}$$

The data for this model are implemented in the spreadsheet shown in Figure 3.17 (and in the file Fig3-17.xls on your data disk). The coefficients that appear in the objective function are entered in the range B10 through D11. The coefficients for the LHS

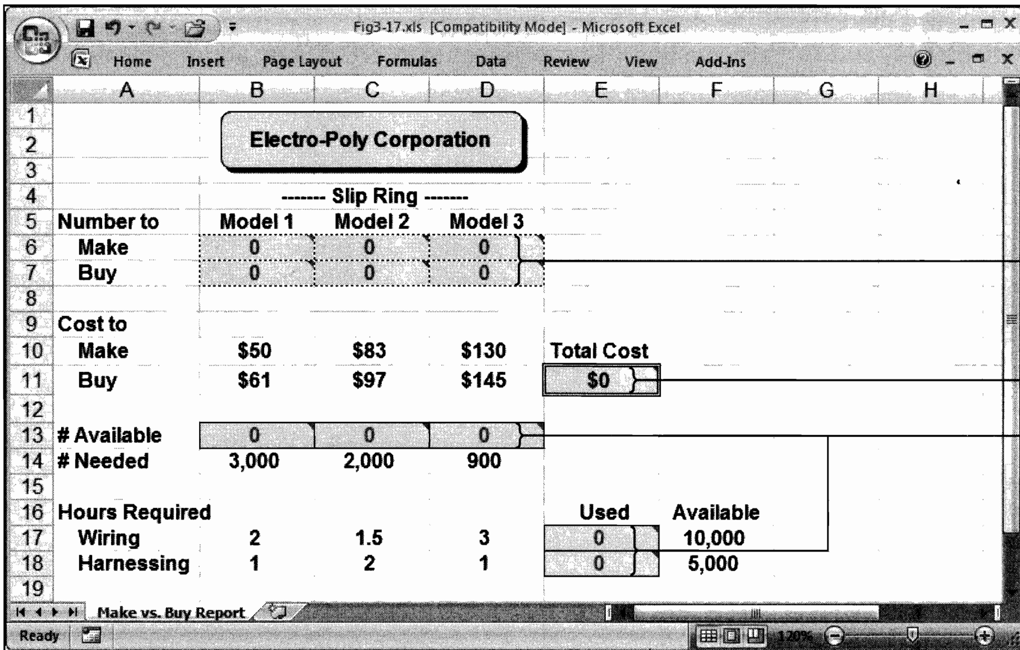


FIGURE 3.17

Spreadsheet model for Electro-Poly's make vs. buy problem

Key Cell Formulas

Cell	Formula	Copied to
B13	=B6+B7	C13:D13
E11	=SUMPRODUCT(B10:D11,B6:D7)	--
E17	=SUMPRODUCT(B17:D17,$B$6:$D$6)	E18

formulas for the wiring and harnessing constraints are entered in cells B17 through D18, and the corresponding RHS values are entered in cells F17 and F18. Because the LHS formulas for the demand constraints involve simply summing the decision variables, we do not need to list the coefficients for these constraints in the spreadsheet. The RHS values for the demand constraints are entered in cells B14 through D14.

Cells B6 through D7 are reserved to represent the six variables in our algebraic model. So, the objective function could be entered in cell E11 as:

Formula for cell E11: =B10*B6+C10*C6+D10*D6+B11*B7+C11*C7+D11*D7

In this formula, the values in the range B6 through D7 are multiplied by the corresponding values in the range B10 through D11; these individual products are then added together. Therefore, the formula is simply the sum of a collection of products—or a *sum of products*. It turns out that this formula can be implemented in an equivalent (and easier) way as:

Equivalent formula for cell E11: =SUMPRODUCT(B10:D11,B6:D7)

The preceding formula takes the values in the range B10 through D11, multiplies them by the corresponding values in the range B6 through D7, and adds (or sums) these products. The SUMPRODUCT( ) function greatly simplifies the implementation of many formulas required in optimization problems and will be used extensively throughout this book.

Because the LHS of the demand constraint for model 1 slip rings involves adding variables M_1 and B_1, this constraint is implemented in cell B13 by adding the two cells in the spreadsheet that correspond to these variables—cells B6 and B7:

Formula for cell B13: =B6+B7
(Copy to C13 through D13.)

The formula in cell B13 is then copied to cells C13 and D13 to implement the LHS formulas for the constraints for model 2 and model 3 slip rings.

The coefficients for the wiring and harnessing constraints are entered in cells B17 through D18. The LHS formula for the wiring constraint is implemented in cell E17 as:

Formula for cell E17: =SUMPRODUCT(B17:D17,$B$6:$D$6)
(Copy to cell E18.)

This formula is then copied to cell E18 to implement the LHS formula for the harnessing constraint. (In the preceding formula, the dollar signs denote absolute cell references. An **absolute cell reference** will not change if the formula containing the reference is copied to another location.)

3.8.5 SOLVING THE MODEL

To solve this model, we need to specify the set cell, variable cells, and constraint cells identified in Figure 3.17, just as we did earlier in the Blue Ridge Hot Tubs example. Figure 3.18 shows the Solver parameters required to solve Electro-Poly's make vs. buy problem.

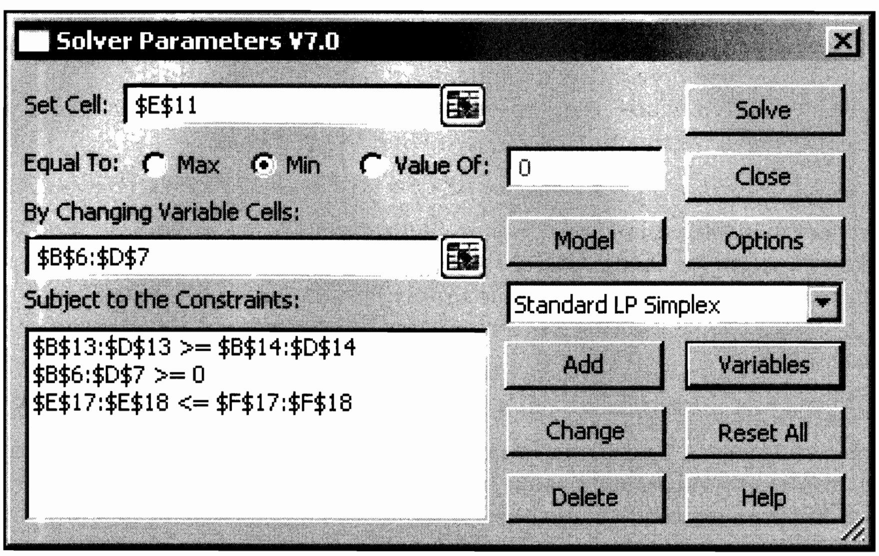
After we click the Solve button in the Solver Parameters dialog box, Solver finds the optimal solution shown in Figure 3.19.

3.8.6 ANALYZING THE SOLUTION

The optimal solution shown in Figure 3.19 indicates that Electro-Poly should make (in-house) 3,000 model 1 slip rings, 550 model 2 slip rings, and 900 model 3 slip rings (that is, $M_1 = 3{,}000$, $M_2 = 550$, $M_3 = 900$). Additionally, it should buy 1,450 model 2 slip rings from its competitor (that is, $B_1 = 0$, $B_2 = 1{,}450$, $B_3 = 0$). This solution allows Electro-Poly

FIGURE 3.18

Solver parameters for the make vs. buy problem



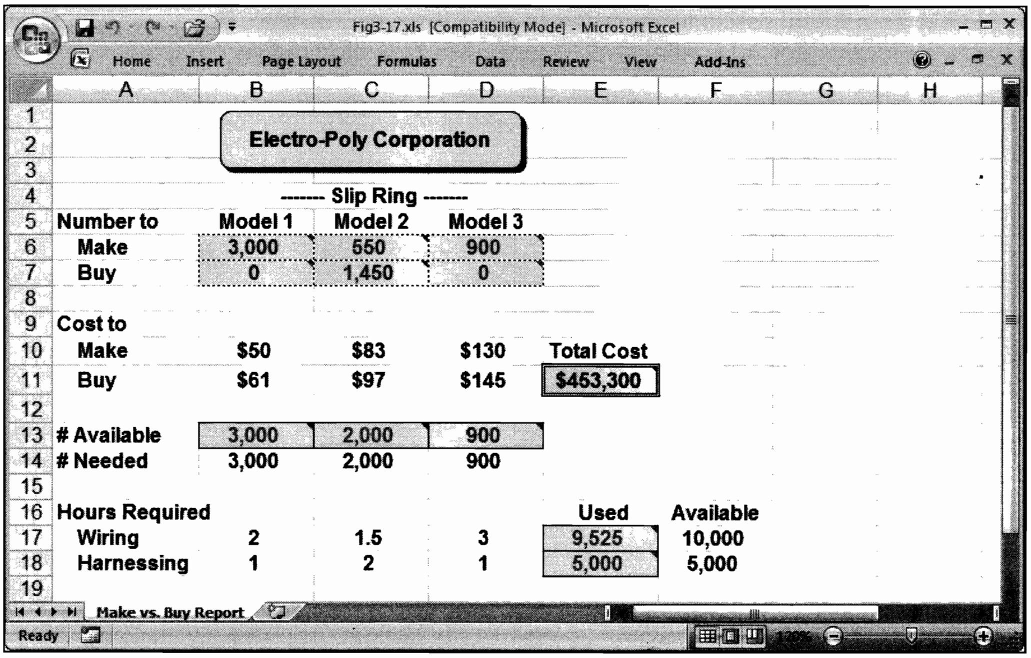


Fig3-17.xls [Compatibility Mode] - Microsoft Excel

	A	B	C	D	E	F
2		Electro-Poly Corporation				
4		------- Slip Ring -------				
5	Number to	Model 1	Model 2	Model 3		
6	Make	3,000	550	900		
7	Buy	0	1,450	0		
9	Cost to					
10	Make	$50	$83	$130	Total Cost	
11	Buy	$61	$97	$145	$453,300	
13	# Available	3,000	2,000	900		
14	# Needed	3,000	2,000	900		
16	Hours Required				Used	Available
17	Wiring	2	1.5	3	9,525	10,000
18	Harnessing	1	2	1	5,000	5,000

Make vs. Buy Report

FIGURE 3.19

Optimal solution to Electro-Poly's make vs. buy problem

to fill the customer order at a minimum cost of $453,300. This solution uses 9,525 of the 10,000 hours of available wiring capacity and all 5,000 hours of the harnessing capacity.

At first glance, this solution might seem a bit surprising. Electro-Poly has to pay $97 for each model 2 slip ring that it purchases from its competitor. This represents a $14 premium over its in-house cost of $83. On the other hand, Electro-Poly has to pay a premium of $11 over its in-house cost to purchase model 1 slip rings from its competitor. It seems as if the optimal solution would be to purchase model 1 slip rings from its competitor rather than model 2 slip rings because the additional cost premium for model 1 slip rings is smaller. However, this argument fails to consider the fact that each model 2 slip ring produced in-house uses twice as much of the company's harnessing capacity as does each model 1 slip ring. Making more model 2 slip rings in-house would deplete the company's harnessing capacity more quickly, and would require buying an excessive number of model 1 slip rings from the competitor. Fortunately, the LP technique automatically considers such trade-offs in determining the optimal solution to the problem.

3.9 An Investment Problem

There are numerous problems in the area of finance to which we can apply various optimization techniques. These problems often involve attempting to maximize the return on an investment while meeting certain cash flow requirements and risk constraints. Alternatively, we might want to minimize the risk on an investment while maintaining a certain level of return. We'll consider one such problem here and discuss several other financial engineering problems throughout this text.

Brian Givens is a financial analyst for Retirement Planning Services, Inc. who specializes in designing retirement income portfolios for retirees using corporate bonds. He has just completed a consultation with a client who expects to have

$750,000 in liquid assets to invest when she retires next month. Brian and his client agreed to consider upcoming bond issues from the following six companies:

Company	Return	Years to Maturity	Rating
Acme Chemical	8.65%	11	1-Excellent
DynaStar	9.50%	10	3-Good
Eagle Vision	10.00%	6	4-Fair
MicroModeling	8.75%	10	1-Excellent
OptiPro	9.25%	7	3-Good
Sabre Systems	9.00%	13	2-Very Good

The column labeled "Return" in this table represents the expected annual yield on each bond, the column labeled "Years to Maturity" indicates the length of time over which the bonds will be payable, and the column labeled "Rating" indicates an independent underwriter's assessment of the quality or risk associated with each issue.

Brian believes that all of the companies are relatively safe investments. However, to protect his client's income, Brian and his client agreed that no more than 25% of her money should be invested in any one investment and at least half of her money should be invested in long-term bonds that mature in ten or more years. Also, even though DynaStar, Eagle Vision, and OptiPro offer the highest returns, it was agreed that no more than 35% of the money should be invested in these bonds because they also represent the highest risks (i.e., they were rated lower than "very good").

Brian needs to determine how to allocate his client's investments to maximize her income while meeting their agreed-upon investment restrictions.

3.9.1 DEFINING THE DECISION VARIABLES

In this problem, Brian must decide how much money to invest in each type of bond. Because there are six different investment alternatives, we need the following six decision variables:

X_1 = amount of money to invest in Acme Chemical

X_2 = amount of money to invest in DynaStar

X_3 = amount of money to invest in Eagle Vision

X_4 = amount of money to invest in MicroModeling

X_5 = amount of money to invest in OptiPro

X_6 = amount of money to invest in Sabre Systems

3.9.2 DEFINING THE OBJECTIVE FUNCTION

The objective in this problem is to maximize the investment income for Brian's client. Because each dollar invested in Acme Chemical (X_1) earns 8.65% annually, each dollar invested in DynaStar (X_2) earns 9.50%, and so on, the objective function for the problem is expressed as:

MAX: $.0865X_1 + .095X_2 + .10X_3 + .0875X_4 + .0925X_5 + .09X_6$ } total annual return

3.9.3 DEFINING THE CONSTRAINTS

Again, there are several constraints that apply to this problem. First, we must ensure that exactly $750,000 is invested. This is accomplished by the following constraint:

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 750{,}000$$

Next, we must ensure that no more than 25% of the total is invested in any one investment. Twenty-five percent of $750,000 is $187,500. Therefore, Brian can put no more than $187,500 into any one investment. The following constraints enforce this restriction:

$$X_1 \leq 187{,}500$$
$$X_2 \leq 187{,}500$$
$$X_3 \leq 187{,}500$$
$$X_4 \leq 187{,}500$$
$$X_5 \leq 187{,}500$$
$$X_6 \leq 187{,}500$$

Because the bonds for Eagle Vision (X_3) and OptiPro (X_5) are the only ones that mature in fewer than 10 years, the following constraint ensures that at least half the money ($375,000) is placed in investments maturing in ten or more years:

$$X_1 + X_2 + X_4 + X_6 \geq 375{,}000$$

Similarly, the following constraint ensures that no more than 35% of the money ($262,500) is placed in the bonds for DynaStar (X_2), Eagle Vision (X_3), and OptiPro (X_5):

$$X_2 + X_3 + X_5 \leq 262{,}500$$

Finally, because none of the variables in the model can assume a value of less than zero, we also need the following nonnegativity condition:

$$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$$

3.9.4 IMPLEMENTING THE MODEL

The LP model for the Retirement Planning Services, Inc. investment problem is summarized as:

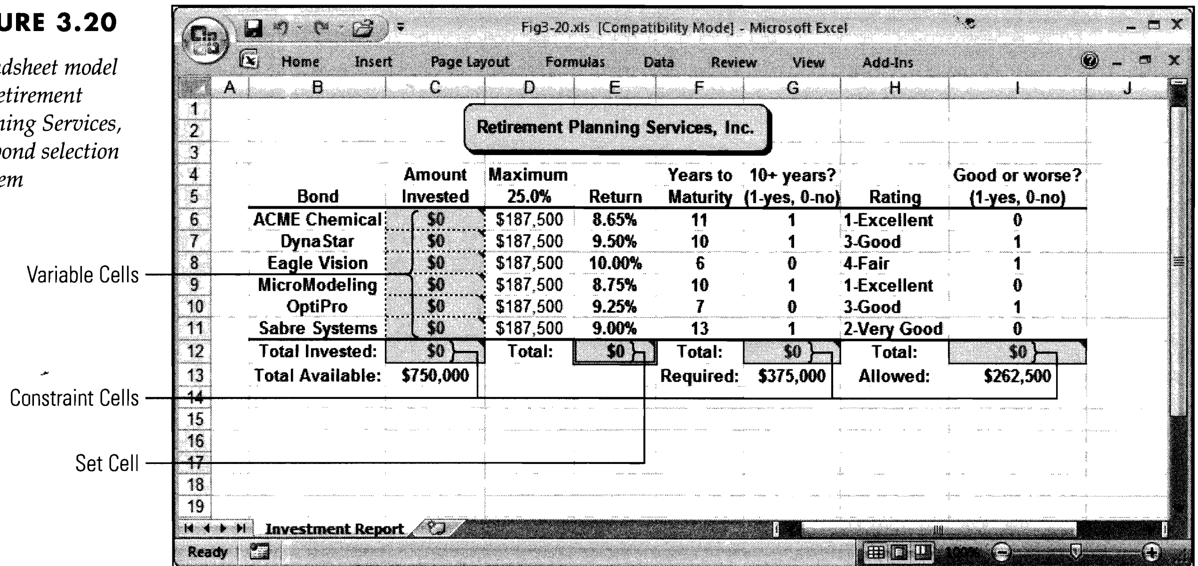
MAX: $.0865X_1 + .095X_2 + .10X_3 + .0875X_4 + .0925X_5 + .09X_6$ } total annual return

Subject to:

$X_1 \leq 187{,}500$	} 25% restriction per investment
$X_2 \leq 187{,}500$	} 25% restriction per investment
$X_3 \leq 187{,}500$	} 25% restriction per investment
$X_4 \leq 187{,}500$	} 25% restriction per investment
$X_5 \leq 187{,}500$	} 25% restriction per investment
$X_6 \leq 187{,}500$	} 25% restriction per investment
$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 750{,}000$	} total amount invested
$X_1 + X_2 + X_4 + X_6 \geq 375{,}000$	} long-term investment
$X_2 + X_3 + X_5 \leq 262{,}500$	} higher-risk investment
$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$	} nonnegativity conditions

FIGURE 3.20

Spreadsheet model for Retirement Planning Services, Inc. bond selection problem



Bond	Amount Invested	Maximum 25.0%	Return	Years to Maturity	10+ years? (1-yes, 0-no)	Rating	Good or worse? (1-yes, 0-no)
ACME Chemical	$0	$187,500	8.65%	11	1	1-Excellent	0
DynaStar	$0	$187,500	9.50%	10	1	3-Good	1
Eagle Vision	$0	$187,500	10.00%	6	0	4-Fair	1
MicroModeling	$0	$187,500	8.75%	10	1	1-Excellent	0
OptiPro	$0	$187,500	9.25%	7	0	3-Good	1
Sabre Systems	$0	$187,500	9.00%	13	1	2-Very Good	0
Total Invested:	$0	Total:	$0	Total:	$0	Total:	$0
Total Available:	$750,000			Required:	$375,000	Allowed:	$262,500

Key Cell Formulas

Cell	Formula	Copied to
C12	=SUM(C6:C11)	--
E12	=SUMPRODUCT(E6:E11,$C$6:$C$11)	G12 and I12

A convenient way of implementing this model is shown in Figure 3.20 (file Fig3-20.xls on your data disk). Each row in this spreadsheet corresponds to one of the investment alternatives. Cells C6 through C11 correspond to the decision variables for the problem ($X_1, \ldots, X_6$). The maximum value that each of these cells can take on is listed in cells D6 through D11. These values correspond to the RHS values for the first six constraints. The sum of cells C6 through C11 is computed in cell C12 as follows, and will be restricted to equal the value shown in cell C13:

Formula for cell C12: =SUM(C6:C11)

The annual returns for each investment are listed in cells E6 through E11. The objective function is then implemented conveniently in cell E12 as follows:

Formula for cell E12: =SUMPRODUCT(E6:E11,$C$6:$C$11)

The values in cells G6 through G11 indicate which of these rows correspond to "long-term" investments. Note that the use of ones and zeros in this column makes it convenient to compute the sum of the cells C6, C7, C9, and C11 (representing X_1, X_2, X_4, and X_6) representing the LHS of the "long-term" investment constraint. This is done in cell G12 as follows:

Formula for cell G12: =SUMPRODUCT(G6:G11,$C$6:$C$11)

Similarly, the zeros and ones in cells I6 through I11 indicate the higher-risk investments and allow us to implement the LHS of the "higher-risk investment" constraint as follows:

Formula for cell I12: =SUMPRODUCT(I6:I11,$C$6:$C$11)

Note that the use of zeros and ones in columns G and I to compute the sums of selected decision variables is a very useful modeling technique that makes it easy for the user to change the variables being included in the sums. Also note that the formula for the objective in cell E12 could be copied to cells G12 and I12 to implement LHS formulas for these constraint cells.

3.9.5 SOLVING THE MODEL

To solve this model, we need to specify the set cell, variable cells, and constraint cells identified in Figure 3.20. Figure 3.21 shows the Solver parameters required to solve this problem. After we click the Solve button in the Solver Parameters dialog box, Solver finds the optimal solution shown in Figure 3.22.

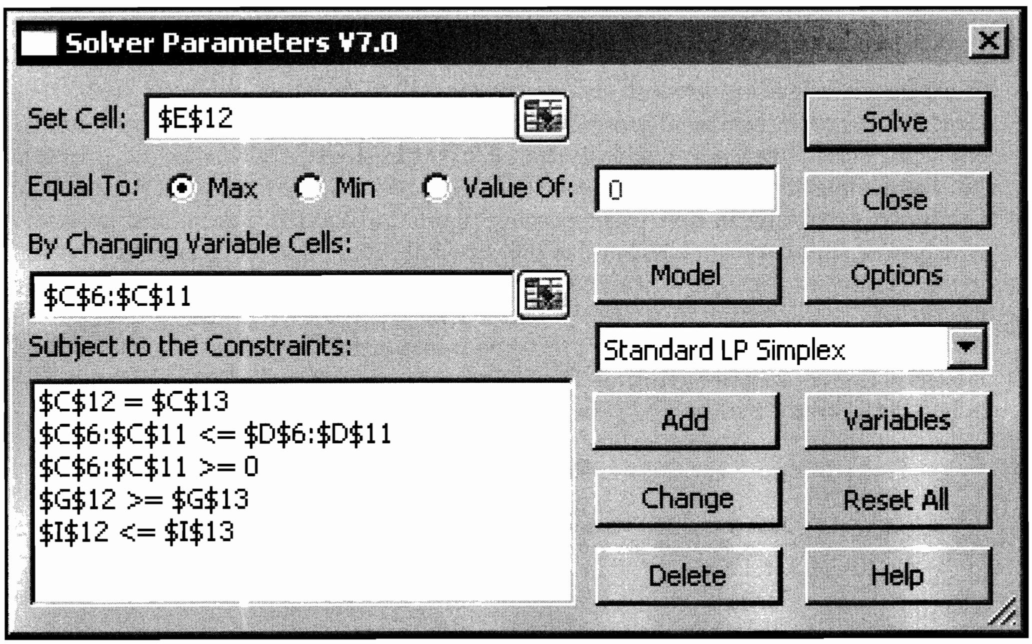
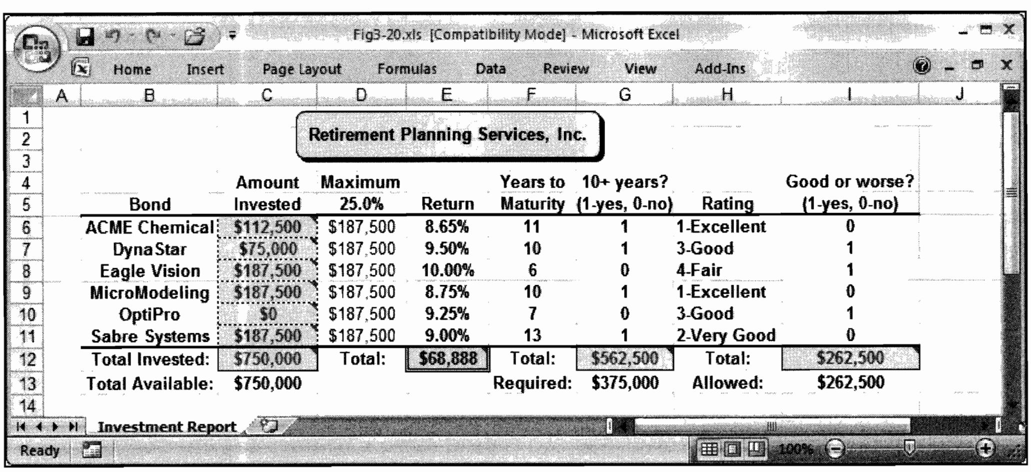


FIGURE 3.21

Solver parameters for the bond selection problem



Bond	Amount Invested	Maximum 25.0%	Return	Years to Maturity	10+ years? (1-yes, 0-no)	Rating	Good or worse? (1-yes, 0-no)
ACME Chemical	$112,500	$187,500	8.65%	11	1	1-Excellent	0
DynaStar	$75,000	$187,500	9.50%	10	1	3-Good	1
Eagle Vision	$187,500	$187,500	10.00%	6	0	4-Fair	1
MicroModeling	$187,500	$187,500	8.75%	10	1	1-Excellent	0
OptiPro	$0	$187,500	9.25%	7	0	3-Good	1
Sabre Systems	$187,500	$187,500	9.00%	13	1	2-Very Good	0
Total Invested:	$750,000	Total:	$68,888	Total:	$562,500	Total:	$262,500
Total Available:	$750,000			Required:	$375,000	Allowed:	$262,500

FIGURE 3.22

Optimal solution to the bond selection problem

3.9.6 ANALYZING THE SOLUTION

The solution shown in Figure 3.22 indicates that the optimal investment plan places \$112,500 in Acme Chemical (X_1), \$75,000 in DynaStar (X_2), \$187,500 in Eagle Vision (X_3), \$187,500 in MicroModeling (X_4), \$0 in OptiPro (X_5), and \$187,500 in Sabre Systems (X_6). It is interesting to note that more money is being invested in Acme Chemical than DynaStar and OptiPro even though the return on Acme Chemical is lower than on the returns for DynaStar and OptiPro. This is because DynaStar and OptiPro are both "higher-risk" investments and the 35% limit on "higher-risk" investments is a binding constraint (or is met as a strict equality in the optimal solution). Thus, the optimal solution could be improved if we could put more than 35% of the money into the higher-risk investments.

3.10 A Transportation Problem

Many transportation and logistics problems businesses face fall into a category of problems known as network flow problems. We will consider one such example here and study this area in more detail in Chapter 5.

Tropicsun is a leading grower and distributor of fresh citrus products with three large citrus groves scattered around central Florida in the cities of Mt. Dora, Eustis, and Clermont. Tropicsun currently has 275,000 bushels of citrus at the grove in Mt. Dora, 400,000 bushels at the grove in Eustis, and 300,000 bushels at the grove in Clermont. Tropicsun has citrus processing plants in Ocala, Orlando, and Leesburg with processing capacities to handle 200,000, 600,000, and 225,000 bushels, respectively. Tropicsun contracts with a local trucking company to transport its fruit from the groves to the processing plants. The trucking company charges a flat rate for every mile that each bushel of fruit must be transported. Each mile a bushel of fruit travels is known as a bushel-mile. The following table summarizes the distances (in miles) between the groves and processing plants:

	Distances (in miles) Between Groves and Plants		
Grove	**Ocala**	**Orlando**	**Leesburg**
Mt. Dora	21	50	40
Eustis	35	30	22
Clermont	55	20	25

Tropicsun wants to determine how many bushels to ship from each grove to each processing plant to minimize the total number of bushel-miles the fruit must be shipped.

3.10.1 DEFINING THE DECISION VARIABLES

In this situation, the problem is to determine how many bushels of fruit should be shipped from each grove to each processing plant. The problem is summarized graphically in Figure 3.23.

The circles (or nodes) in Figure 3.23 correspond to the different groves and processing plants in the problem. Note that a number has been assigned to each node. The arrows (or arcs) connecting the various groves and processing plants represent different shipping routes. The decision problem faced by Tropicsun is to determine how many bushels of fruit to ship on each of these routes. Thus, one decision variable is associated with each of the arcs in Figure 3.23. We can define these variables in general as:

X_{ij} = number of bushels to ship from node i to node j

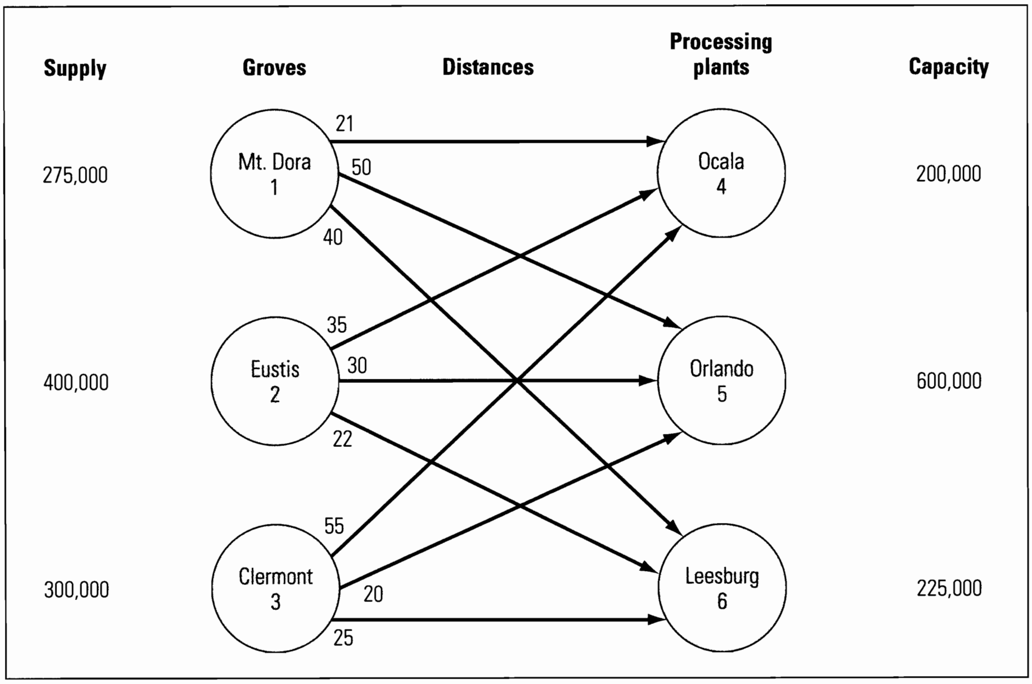


FIGURE 3.23

Diagram for the Tropicsun transportation problem

Specifically, the nine decision variables are:

X_{14} = number of bushels to ship from Mt. Dora (node 1) to Ocala (node 4)
X_{15} = number of bushels to ship from Mt. Dora (node 1) to Orlando (node 5)
X_{16} = number of bushels to ship from Mt. Dora (node 1) to Leesburg (node 6)
X_{24} = number of bushels to ship from Eustis (node 2) to Ocala (node 4)
X_{25} = number of bushels to ship from Eustis (node 2) to Orlando (node 5)
X_{26} = number of bushels to ship from Eustis (node 2) to Leesburg (node 6)
X_{34} = number of bushels to ship from Clermont (node 3) to Ocala (node 4)
X_{35} = number of bushels to ship from Clermont (node 3) to Orlando (node 5)
X_{36} = number of bushels to ship from Clermont (node 3) to Leesburg (node 6)

3.10.2 DEFINING THE OBJECTIVE FUNCTION

The goal in this problem is to determine how many bushels to ship from each grove to each processing plant while minimizing the total distance (or total number of bushel-miles) the fruit must travel. The objective function for this problem is represented by:

$$\text{MIN:} \quad 21X_{14} + 50X_{15} + 40X_{16} + 35X_{24} + 30X_{25} + 22X_{26} + 55X_{34} + 20X_{35} + 25X_{36}$$

The term $21X_{14}$ in this function reflects the fact that each bushel shipped from Mt. Dora (node 1) to Ocala (node 4) must travel 21 miles. The remaining terms in the function express similar relationships for the other shipping routes.

3.10.3 DEFINING THE CONSTRAINTS

Two physical constraints apply to this problem. First, there is a limit on the amount of fruit that can be shipped to each processing plant. Tropicsun can ship no more than

200,000, 600,000, and 225,000 bushels to Ocala, Orlando, and Leesburg, respectively. These restrictions are reflected by the following constraints:

$$\begin{aligned} X_{14} + X_{24} + X_{34} &\le 200{,}000 && \text{\} capacity restriction for Ocala} \\ X_{15} + X_{25} + X_{35} &\le 600{,}000 && \text{\} capacity restriction for Orlando} \\ X_{16} + X_{26} + X_{36} &\le 225{,}000 && \text{\} capacity restriction for Leesburg} \end{aligned}$$

The first constraint indicates that the total bushels shipped to Ocala (node 4) from Mt. Dora (node 1), Eustis (node 2), and Clermont (node 3) must be less than or equal to Ocala's capacity of 200,000 bushels. The other two constraints have similar interpretations for Orlando and Leesburg. Notice that the total processing capacity at the plants (1,025,000 bushels) exceeds the total supply of fruit at the groves (975,000 bushels). Therefore, these constraints are "less than or equal to" constraints because not all the available capacity will be used.

The second set of constraints ensures that the supply of fruit at each grove is shipped to a processing plant. That is, all of the 275,000, 400,000, and 300,000 bushels at Mt. Dora, Eustis, and Clermont, respectively, must be processed somewhere. This is accomplished by the following constraints:

$$\begin{aligned} X_{14} + X_{15} + X_{16} &= 275{,}000 && \text{\} supply available at Mt. Dora} \\ X_{24} + X_{25} + X_{26} &= 400{,}000 && \text{\} supply available at Eustis} \\ X_{34} + X_{35} + X_{36} &= 300{,}000 && \text{\} supply available at Clermont} \end{aligned}$$

The first constraint indicates that the total amount shipped from Mt. Dora (node 1) to the plants in Ocala (node 4), Orlando (node 5), and Leesburg (node 6) must equal the total amount available at Mt. Dora. This constraint indicates that all the fruit available at Mt. Dora must be shipped somewhere. The other two constraints play similar roles for Eustis and Clermont.

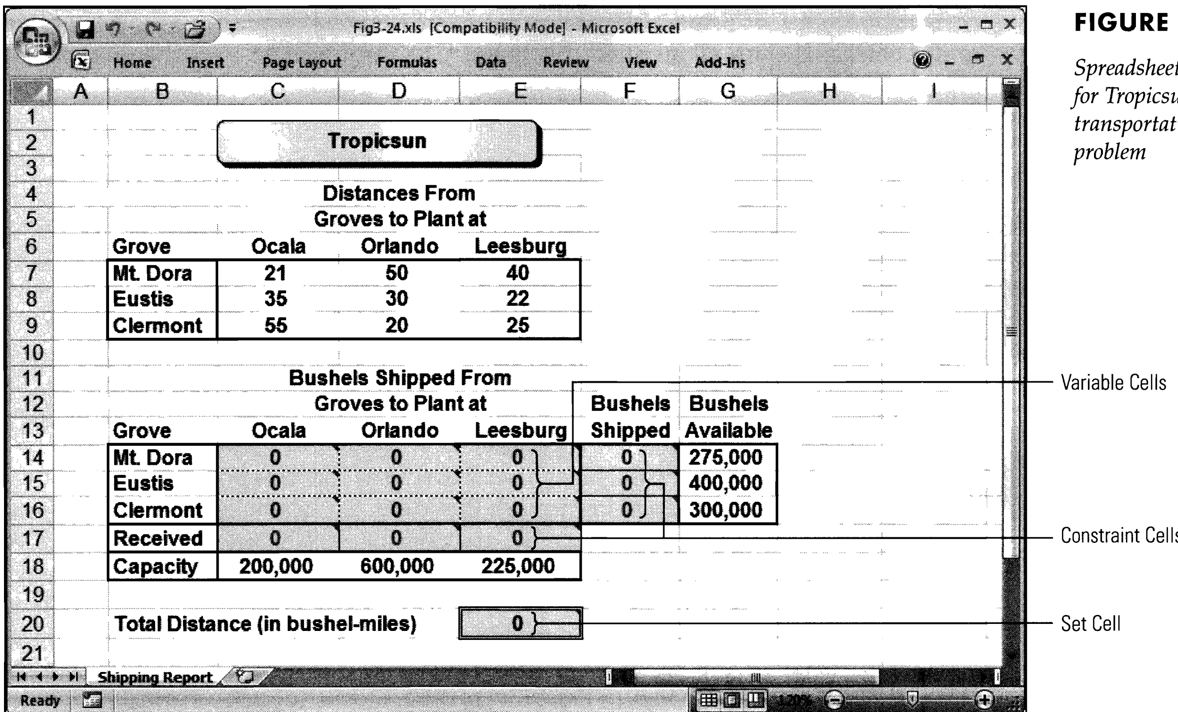
3.10.4 IMPLEMENTING THE MODEL

The LP model for Tropicsun's fruit transportation problem is summarized as:

$$\begin{aligned} \text{MIN:} \quad & \left.\begin{array}{l} 21X_{14} + 50X_{15} + 40X_{16} + \\ 35X_{24} + 30X_{25} + 22X_{26} + \\ 55X_{34} + 20X_{35} + 25X_{36} \end{array}\right\} && \begin{array}{l}\text{total distance fruit is shipped} \\ \text{(in bushel-miles)}\end{array} \\ \text{Subject to:} \quad & X_{14} + X_{24} + X_{34} \le 200{,}000 && \text{\} capacity restriction for Ocala} \\ & X_{15} + X_{25} + X_{35} \le 600{,}000 && \text{\} capacity restriction for Orlando} \\ & X_{16} + X_{26} + X_{36} \le 225{,}000 && \text{\} capacity restriction for Leesburg} \\ & X_{14} + X_{15} + X_{16} = 275{,}000 && \text{\} supply available at Mt. Dora} \\ & X_{24} + X_{25} + X_{26} = 400{,}000 && \text{\} supply available at Eustis} \\ & X_{34} + X_{35} + X_{36} = 300{,}000 && \text{\} supply available at Clermont} \\ & X_{ij} \ge 0, \text{ for all } i \text{ and } j && \text{\} nonnegativity conditions} \end{aligned}$$

The last constraint, as in previous models, indicates that all the decision variables must be nonnegative.

A convenient way to implement this model is shown in Figure 3.24 (and in the file Fig3-24.xls on your data disk). In this spreadsheet, the distances between each grove and plant are summarized in a tabular format in cells C7 through E9. Cells C14 through E16 are reserved for representing the number of bushels of fruit to ship from each grove to each processing plant. Notice that these nine cells correspond directly to the nine decision variables in the algebraic formulation of the model.



Key Cell Formulas

Cell	Formula	Copied to
C17	=SUM(C14:C16)	D17:E17
F14	=SUM(C14:E14)	F15:F16
E20	=SUMPRODUCT(C7:E9,C14:E16)	--

FIGURE 3.24

Spreadsheet model for Tropicsun's transportation problem

The LHS formulas for the three capacity constraints in the model are implemented in cells C17, D17, and E17 in the spreadsheet. To do this, the following formula is entered in cell C17 and copied to cells D17 and E17:

Formula for cell C17: =SUM(C14:C16)
(Copy to D17 and E17.)

These cells represent the total bushels of fruit being shipped to the plants in Ocala, Orlando, and Leesburg, respectively. Cells C18 through E18 contain the RHS values for these constraint cells.

The LHS formulas for the three supply constraints in the model are implemented in cells F14, F15, and F16 as:

Formula for cell F14: =SUM(C14:E14)
(Copy to F15 and F16.)

These cells represent the total bushels of fruit being shipped from the groves at Mt. Dora, Eustis, and Clermont, respectively. Cells G14 through G16 contain the RHS values for these constraint cells.

Finally, the objective function for this model is entered in cell E20 as:

Formula for cell E20: =SUMPRODUCT(C7:E9,C14:E16)

The SUMPRODUCT( ) function multiplies each element in the range C7 through E9 by the corresponding element in the range C14 through E16 and then sums the individual products.

3.10.5 HEURISTIC SOLUTION FOR THE MODEL

To appreciate what Solver is accomplishing, let's consider how we might try to solve this problem manually using a heuristic. A **heuristic** is a rule of thumb for making decisions that might work well in some instances, but is not guaranteed to produce optimal solutions or decisions. One heuristic we can apply to solve Tropicsun's problem is to always ship as much as possible along the next available path with the shortest distance (or least cost). Using this heuristic, we solve the problem as follows:

1. Because the shortest available path between any grove and processing plant is between Clermont and Orlando (20 miles), we first ship as much as possible through this route. The maximum we can ship through this route is the smaller of the supply at Clermont (300,000 bushels) or the capacity at Orlando (600,000 bushels). So we would ship 300,000 bushels from Clermont to Orlando. This depletes the supply at Clermont.
2. The next shortest available route occurs between Mt. Dora and Ocala (21 miles). The maximum we can ship through this route is the smaller of the supply at Mt. Dora (275,000 bushels) or the capacity at Ocala (200,000 bushels). So we would ship 200,000 bushels from Mt. Dora to Ocala. This depletes the capacity at Ocala.
3. The next shortest available route occurs between Eustis and Leesburg (22 miles). The maximum we can ship through this route is the smaller of the supply at Eustis (400,000 bushels) or the capacity at Leesburg (225,000 bushels). So we would ship 225,000 bushels from Eustis to Leesburg. This depletes the capacity at Leesburg.
4. The next shortest available route occurs between Eustis and Orlando (30 miles). The maximum we can ship through this route is the smaller of the remaining supply at Eustis (175,000 bushels) or the remaining capacity at Orlando (300,000 bushels). So we would ship 175,000 bushels from Eustis to Orlando. This depletes the supply at Eustis.
5. The only remaining route occurs between Mt. Dora and Orlando (because the processing capacities at Ocala and Leesburg have both been depleted). This distance is 50 miles. The maximum we can ship through this route is the smaller of the remaining supply at Mt. Dora (75,000 bushels) and the remaining capacity at Orlando (125,000 bushels). So we would ship the final 75,000 bushels at Mt. Dora to Orlando. This depletes the supply at Mt. Dora.

As shown in Figure 3.25, the solution identified with this heuristic involves shipping the fruit a total of 24,150,000 bushel-miles. All the bushels available at each grove have been shipped to the processing plants and none of the capacities at the processing plants have been exceeded. Therefore, this is a *feasible* solution to the problem. And the logic used to find this solution might lead us to believe it is a reasonably good solution—but is it the *optimal* solution? Is there no other feasible solution to this problem that can make the total distance the fruit has to travel less than 24,150,000 bushel-miles?

3.10.6 SOLVING THE MODEL

To find the optimal solution to this model, we must indicate to Solver the set cell, variable cells, and constraint cells identified in Figure 3.24. Figure 3.26 shows the Solver parameters required to solve this problem. The optimal solution is shown in Figure 3.27.

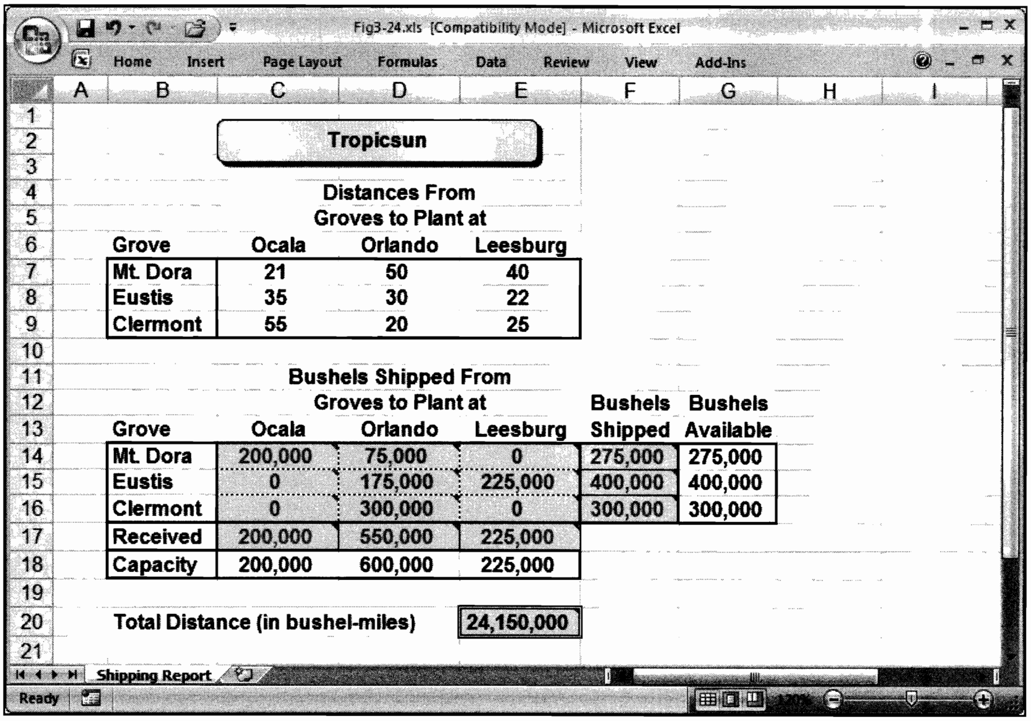


FIGURE 3.25

A heuristic solution to the transportation problem

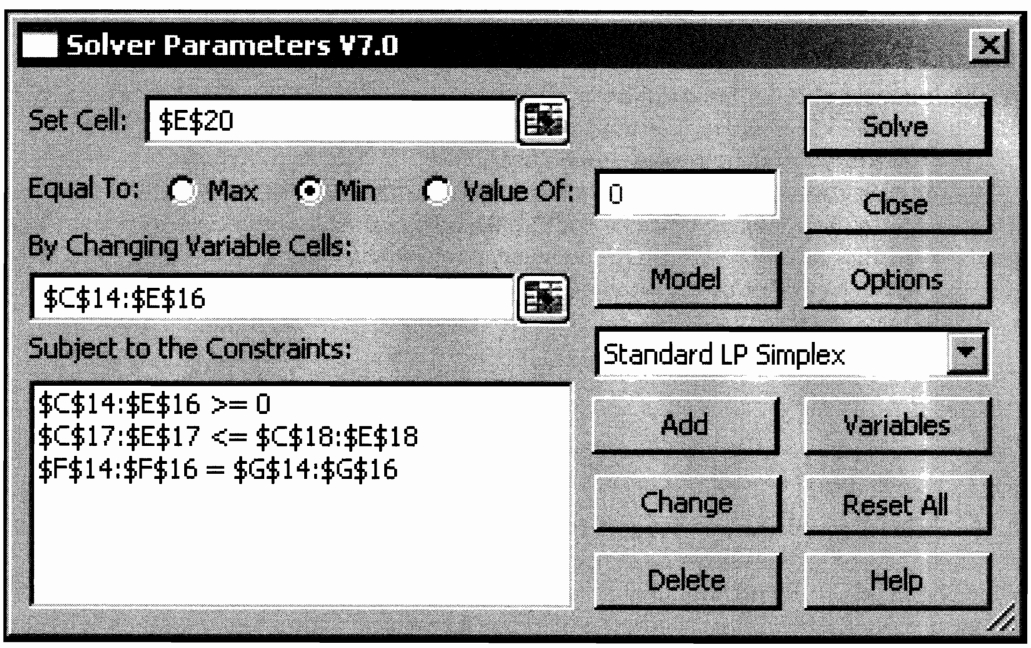


FIGURE 3.26

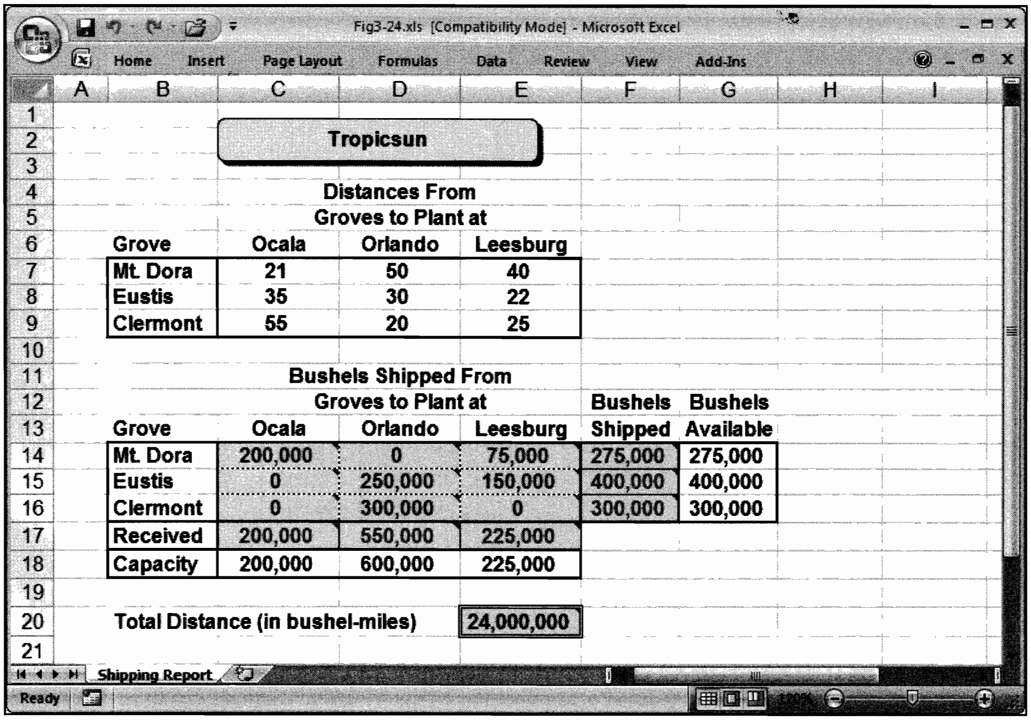
Solver parameters for the transportation problem

3.10.7 ANALYZING THE SOLUTION

The optimal solution in Figure 3.27 indicates that 200,000 bushels should be shipped from Mt. Dora to Ocala ($X_{14} = 200{,}000$) and 75,000 bushels should be shipped from Mt. Dora to Leesburg ($X_{16} = 75{,}000$). Of the 400,000 bushels available at the grove in Eustis, 250,000 bushels should be shipped to Orlando for processing ($X_{25} = 250{,}000$) and

FIGURE 3.27

Optimal solution to Tropicsun's transportation problem



Tropicsun

Grove	Distances From Groves to Plant at Ocala	Orlando	Leesburg
Mt. Dora	21	50	40
Eustis	35	30	22
Clermont	55	20	25

Grove	Bushels Shipped From Groves to Plant at Ocala	Orlando	Leesburg	Bushels Shipped	Bushels Available
Mt. Dora	200,000	0	75,000	275,000	275,000
Eustis	0	250,000	150,000	400,000	400,000
Clermont	0	300,000	0	300,000	300,000
Received	200,000	550,000	225,000		
Capacity	200,000	600,000	225,000		

Total Distance (in bushel-miles) 24,000,000

150,000 bushels should be shipped to Leesburg ($X_{26} = 150{,}000$). Finally, all 300,000 bushels available in Clermont should be shipped to Orlando ($X_{35} = 300{,}000$). None of the other possible shipping routes will be used.

The solution shown in Figure 3.27 satisfies all the constraints in the model and results in a minimum shipping distance of 24,000,000 bushel-miles, which is better than the heuristic solution identified earlier. Therefore, simple heuristics can solve LP problems sometimes, but as this example illustrates, there is no guarantee that a heuristic solution is the best possible solution.

3.11 A Blending Problem

Many business problems involve determining an optimal mix of ingredients. For example, major oil companies must determine the least costly mix of different crude oils and other chemicals to blend together to produce a certain grade of gasoline. Lawn care companies must determine the least costly mix of chemicals and other products to blend together to produce different types of fertilizer. The following is another example of a common blending problem faced in the U.S. agricultural industry, which annually produces goods valued at approximately $200 billion.

> Agri-Pro is a company that sells agricultural products to farmers in several states. One service it provides to customers is custom feed mixing, whereby a farmer can order a specific amount of livestock feed and specify the amount of corn, grain, and minerals the feed should contain. This is an important service because the proper

feed for various farm animals changes regularly depending on the weather, pasture conditions, and so on.

Agri-Pro stocks bulk amounts of four types of feeds that it can mix to meet a given customer's specifications. The following table summarizes the four feeds, their composition of corn, grain, and minerals, and the cost per pound for each type.

	Percent of Nutrient in			
Nutrient	**Feed 1**	**Feed 2**	**Feed 3**	**Feed 4**
Corn	30%	5%	20%	10%
Grain	10%	30%	15%	10%
Minerals	20%	20%	20%	30%
Cost per Pound	\$0.25	\$0.30	\$0.32	\$0.15

On average, U.S. citizens consume almost 70 pounds of poultry per year. To remain competitive, chicken growers must ensure that they feed the required nutrients to their flocks in the most cost-effective manner. Agri-Pro has just received an order from a local chicken farmer for 8,000 pounds of feed. The farmer wants this feed to contain at least 20% corn, 15% grain, and 15% minerals. What should Agri-Pro do to fill this order at minimum cost?

3.11.1 DEFINING THE DECISION VARIABLES

In this problem, Agri-Pro must determine how much of the various feeds to blend together to meet the customer's requirements at minimum cost. An algebraic formulation of this problem might use the following four decision variables:

$$X_1 = \text{pounds of feed 1 to use in the mix}$$
$$X_2 = \text{pounds of feed 2 to use in the mix}$$
$$X_3 = \text{pounds of feed 3 to use in the mix}$$
$$X_4 = \text{pounds of feed 4 to use in the mix}$$

3.11.2 DEFINING THE OBJECTIVE FUNCTION

The objective in this problem is to fill the customer's order at the lowest possible cost. Because each pound of feed 1, 2, 3, and 4 costs \$0.25, \$0.30, \$0.32, and \$0.15, respectively, the objective function is represented by:

$$\text{MIN:} \quad .25X_1 + .30X_2 + .32X_3 + .15X_4$$

3.11.3 DEFINING THE CONSTRAINTS

Four constraints must be met to fulfill the customer's requirements. First, the customer wants a total of 8,000 pounds of feed. This is expressed by the constraint:

$$X_1 + X_2 + X_3 + X_4 = 8{,}000$$

The customer also wants the order to consist of at least 20% corn. Because each pound of feed 1, 2, 3, and 4 consists of 30%, 5%, 20%, and 10% corn, respectively, the total amount of corn in the mix is represented by:

$$.30X_1 + .05X_2 + .20X_3 + .10X_4$$

To ensure that *corn* constitutes at least 20% of the 8,000 pounds of feed, we set up the following constraint:

$$\frac{.30X_1 + .05X_2 + .20X_3 + .10X_4}{8{,}000} \geq .20$$

Similarly, to ensure that *grain* constitutes at least 15% of the 8,000 pounds of feed, we use the constraint:

$$\frac{.10X_1 + .30X_2 + .15X_3 + .10X_4}{8{,}000} \geq .15$$

Finally, to ensure that *minerals* constitute at least 15% of the 8,000 pounds of feed, we use the constraint:

$$\frac{.20X_1 + .20X_2 + .20X_3 + .30X_4}{8{,}000} \geq .15$$

3.11.4 SOME OBSERVATIONS ABOUT CONSTRAINTS, REPORTING, AND SCALING

We need to make some important observations about the constraints for this model. First, these constraints look somewhat different from the usual linear sum of products. However, these constraints are equivalent to a sum of products. For example, the constraint for the required percentage of corn can be expressed as:

$$\frac{.30X_1 + .05X_2 + .20X_3 + .10X_4}{8000} \geq .20$$

or as:

$$\frac{.30X_1}{8{,}000} + \frac{5.0X_2}{8{,}000} + \frac{.20X_3}{8{,}000} + \frac{.10X_4}{8{,}000} \geq .20$$

or, if you multiply both sides of the inequality by 8,000, as:

$$.30X_1 + .05X_2 + .20X_3 + .10X_4 \geq 1{,}600$$

All these constraints define exactly the same set of feasible values for $X_1, \ldots, X_4$. Theoretically, we should be able to implement and use *any* of these constraints to solve the problem. However, we need to consider a number of practical issues in determining which form of the constraint to implement.

Notice that the LHS formulas for the first and second versions of the constraint represent the *proportion* of corn in the 8,000 pound order, whereas the LHS in the third version of the constraint represents the *total pounds* of corn in the 8,000 pound order. Because we must implement the LHS formula of one of these constraints in the spreadsheet, we need to decide which number to display in the spreadsheet—the *proportion* (or percentage) of corn in the order, or the *total pounds of corn* in the order. If we know one of these values, we can easily set up a formula to calculate the other value. But, when more than one way to implement a constraint exists (as is usually the case), we need to consider what the value of the LHS portion of the constraint means to the user of the spreadsheet so that the results of the model can be reported as clearly as possible.

Another issue to consider involves *scaling* the model so that it can be solved accurately. For example, suppose we decide to implement the LHS formula for the first or second version of the corn constraint given earlier so that the *proportion* of corn in the 8,000 pound feed order appears in the spreadsheet. The coefficients for the variables in these constraints are *very* small values. In either case, the coefficient for X_2 is 0.05/8,000 or 0.000006250.

As Solver tries to solve an LP problem, it must perform intermediate calculations that make the various coefficients in the model larger or smaller. As numbers become extremely large or small, computers often run into storage or representation problems that force them to use approximations of the actual numbers. This opens the door for problems to occur in the accuracy of the results and, in some cases, can prevent the computer from solving the problem at all. So, if some coefficients in the initial model are extremely large or extremely small, it is a good idea to rescale the problem so that all the coefficients are of similar magnitudes.

3.11.5 RESCALING THE MODEL

To illustrate how a problem is rescaled, consider the following equivalent formulation of the Agri-Pro problem:

X_1 = amount of feed 1 *in thousands of pounds* to use in the mix
X_2 = amount of feed 2 *in thousands of pounds* to use in the mix
X_3 = amount of feed 3 *in thousands of pounds* to use in the mix
X_4 = amount of feed 4 *in thousands of pounds* to use in the mix

The objective function and constraints are represented by:

$$\begin{array}{lrll}
\text{MIN:} & 250X_1 + 300X_2 + 320X_3 + 150X_4 & & \} \text{ total cost} \\
\text{Subject to:} & X_1 + X_2 + X_3 + X_4 = & 8 & \} \text{ pounds of feed required} \\
& \dfrac{.30X_1 + .05X_2 + .20X_3 + .10X_4}{8} \geq & 0.20 & \} \text{ min \% of corn required} \\
& \dfrac{.10X_1 + .30X_2 + .15X_3 + .10X_4}{8} \geq & 0.15 & \} \text{ min \% of grain required} \\
& \dfrac{.20X_1 + .20X_2 + .20X_3 + .30X_4}{8} \geq & 0.15 & \} \text{ min \% of minerals required} \\
& X_1, X_2, X_3, X_4 \geq & 0 & \} \text{ nonnegativity conditions}
\end{array}$$

Each unit of X_1, X_2, X_3, and X_4 now represents 1,000 pounds of feed 1, 2, 3, and 4, respectively. So the objective now reflects the fact that each unit (or each 1,000 pounds) of X_1, X_2, X_3, and X_4 costs \$250, \$300, \$320, and \$150, respectively. The constraints have also been adjusted to reflect the fact that the variables now represent thousands of pounds of the different feeds. Notice that the smallest coefficient in the constraints is now $0.05/8 = 0.00625$ and the largest coefficient is 8 (that is, the RHS value for the first constraint). In our original formulation, the smallest coefficient was 0.00000625 and the largest coefficient was 8,000. By rescaling the problem, we dramatically reduced the range between the smallest and largest coefficients in the model.

Automatic Scaling

In solving some earlier problems in this chapter, you might have noticed that the Solver Options dialog box provides an option called Use Automatic Scaling (see Figure 3.14). If you select this option, Solver attempts to rescale the data automatically before solving the problem. Although this option is effective, you should not rely solely on it to solve all scaling problems that occur in your models.

Scaling and Linear Models

When the Standard LP Simplex solving option is selected, Solver conducts several internal tests to verify that the model is truly linear in the objective and constraints. If Solver's tests indicate that the model is not linear, a dialog box appears indicating that the conditions for linearity are not satisfied. The internal tests Solver applies are nearly 100% accurate but sometimes indicate that the model is not linear when, in fact, it is. This often occurs when a model is poorly scaled. If you encounter this message and you are certain that your model is linear, re-solving the model might result in Solver identifying the optimal solution. If this does not work, try reformulating your model so that it is more evenly scaled.

3.11.6 IMPLEMENTING THE MODEL

One way to implement this model in a spreadsheet is shown in Figure 3.28 (and in the file Fig3-28.xls on your data disk). In this spreadsheet, cells B5 through E5 contain the costs of the different types of feeds. The percentage of the different nutrients found in each type of feed is listed in cells B10 through E12.

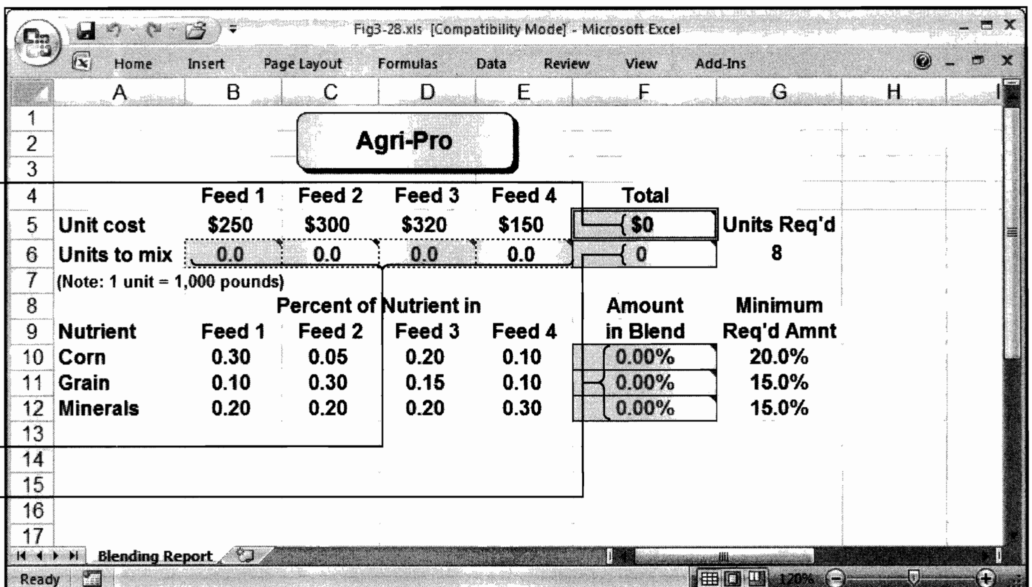
FIGURE 3.28

Spreadsheet model for Agri-Pro's blending problem

Set Cell

Variable Cells

Constraint Cells



Key Cell Formulas

Cell	Formula	Copied to
F5	=SUMPRODUCT(B5:E5,B6:E6)	--
F6	=SUM(B6:E6)	--
F10	=SUMPRODUCT(B10:E10,$B$6:$E$6)/$G$6	F11:F12

Cell G6 contains the total amount of feed (in 1,000s of pounds) required for the order, and the minimum percentage of the three types of nutrients required by the customer order are entered in cells G10 through G12. Notice that the values in column G correspond to the RHS values for the various constraints in the model.

In this spreadsheet, cells B6, C6, D6, and E6 are reserved to represent the decision variables X_1, X_2, X_3, and X_4. These cells ultimately will indicate how much of each type of feed should be mixed together to fill the order. The objective function for the problem is implemented in cell F5 using the formula:

Formula for cell F5: =SUMPRODUCT(B5:E5,B6:E6)

The LHS formula for the first constraint involves calculating the sum of the decision variables. This relationship is implemented in cell F6 as:

Formula for cell F6: =SUM(B6:E6)

The RHS for this constraint is in cell G6. The LHS formulas for the other three constraints are implemented in cells F10, F11, and F12. Specifically, the LHS formula for the second constraint (representing the proportion of corn in the mix) is implemented in cell F10 as:

Formula for cell F10: =SUMPRODUCT(B10:E10,$B$6:$E$6)/$G$6
(Copy to F11 through F12.)

This formula is then copied to cells F11 and F12 to implement the LHS formulas for the remaining two constraints. Again, cells G10 through G12 contain the RHS values for these constraints.

Notice that this model is implemented in a user-friendly way. Each constraint cell has a logical interpretation that communicates important information. For any given values for the variable cells (B6 through E6) totaling 8,000, the constraint cells (F10 through F12) indicate the *actual* percentage of corn, grain, and minerals in the mix.

3.11.7 SOLVING THE MODEL

Figure 3.29 shows the Solver parameters required to solve this problem. The optimal solution is shown in Figure 3.30.

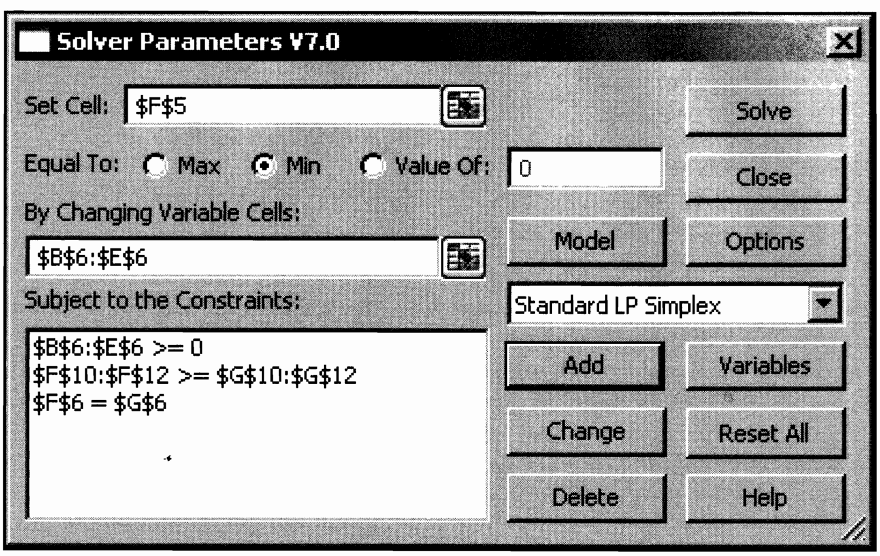
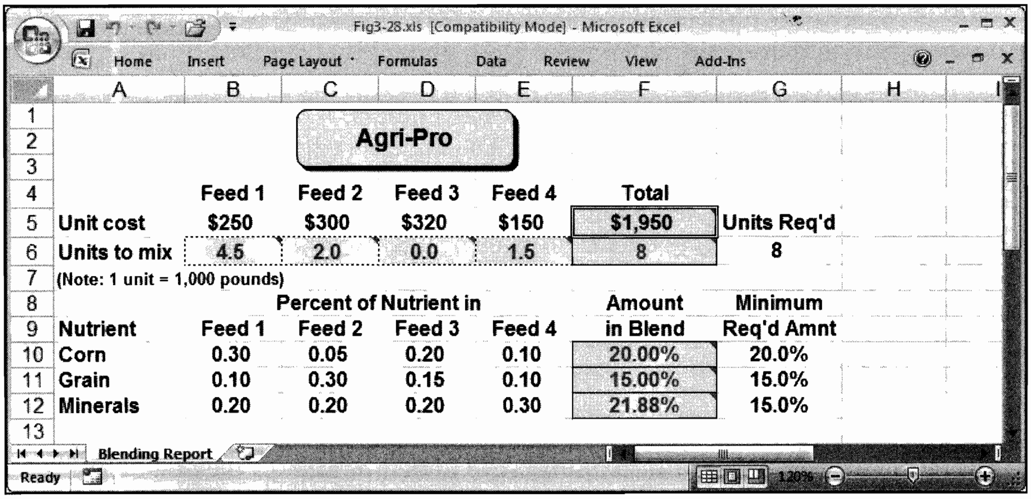


FIGURE 3.29

Solver parameters for the blending problem

FIGURE 3.30

Optimal solution to Agri-Pro's blending problem



	A	B	C	D	E	F	G
1							
2			Agri-Pro				
3							
4		Feed 1	Feed 2	Feed 3	Feed 4	Total	
5	Unit cost	$250	$300	$320	$150	$1,950	Units Req'd
6	Units to mix	4.5	2.0	0.0	1.5	8	8
7	(Note: 1 unit = 1,000 pounds)						
8			Percent of Nutrient in			Amount	Minimum
9	Nutrient	Feed 1	Feed 2	Feed 3	Feed 4	in Blend	Req'd Amnt
10	Corn	0.30	0.05	0.20	0.10	20.00%	20.0%
11	Grain	0.10	0.30	0.15	0.10	15.00%	15.0%
12	Minerals	0.20	0.20	0.20	0.30	21.88%	15.0%
13							

3.11.8 ANALYZING THE SOLUTION

The optimal solution shown in Figure 3.30 indicates that the 8,000 pound feed order is produced at the lowest possible cost by mixing 4,500 pounds of feed 1 ($X_1 = 4.5$) with 2,000 pounds of feed 2 ($X_2 = 2$) and 1,500 pounds of feed 4 ($X_4 = 1.5$). Cell F6 indicates this produces exactly 8,000 pounds of feed. Furthermore, cells F10 through F12 indicate this mix contains 20% corn, 15% grain, and 21.88% minerals. The total cost of producing this mix is $1,950, as indicated by cell F5.

Have You Seen LP at Your Grocery Store?

The next time you are at your local grocery store, make a special trip down the aisle where the pet food is located. On the back of just about any bag of dog or cat food, you should see the following sort of label (taken directly from the author's dog's favorite brand of food):

This product contains:

- At least 21% crude protein
- At least 8% crude fat
- At most 4.5% crude fiber
- At most 12% moisture

In making such statements, the manufacturer guarantees that these nutritional requirements are met by the product. Various ingredients (such as corn, soybeans, meat and bone meal, animal fat, wheat, and rice) are blended to make the product. Most companies are interested in determining the blend of ingredients that satisfies these requirements in the least costly way. Not surprisingly, almost all of the major pet food manufacturing companies use LP extensively in their production process to solve this type of blending problem.

3.12 A Production and Inventory Planning Problem

One of the most fundamental problems facing manufacturing companies is that of planning their production and inventory levels. This process considers demand forecasts and resource constraints for the next several time periods and determines production and inventory levels for each of these time periods so as to meet the anticipated demand in the most economical way. As the following example illustrates, the multiperiod nature of these problems can be handled very conveniently in a spreadsheet to greatly simplify the production planning process.

The Upton Corporation manufactures heavy-duty air compressors for the home and light industrial markets. Upton is presently trying to plan its production and inventory levels for the next six months. Because of seasonal fluctuations in utility and raw material costs, the per unit cost of producing air compressors varies from month to month—as does the demand for air compressors. Production capacity also varies from month to month due to differences in the number of working days, vacations, and scheduled maintenance and training. The following table summarizes the monthly production costs, demands, and production capacity Upton's management expects to face over the next six months.

	Month					
	1	**2**	**3**	**4**	**5**	**6**
Unit Production Cost	$ 240	$ 250	$ 265	$ 285	$ 280	$ 260
Units Demanded	1,000	4,500	6,000	5,500	3,500	4,000
Maximum Production	4,000	3,500	4,000	4,500	4,000	3,500

Given the size of Upton's warehouse, a maximum of 6,000 units can be held in inventory at the end of any month. The owner of the company likes to keep at least 1,500 units in inventory as safety stock to meet unexpected demand contingencies. To maintain a stable workforce, the company wants to produce no less than one half of its maximum production capacity each month. Upton's controller estimates that the cost of carrying a unit in any given month is approximately equal to 1.5% of the unit production cost in the same month. Upton estimates the number of units carried in inventory each month by averaging the beginning and ending inventory for each month.

There are 2,750 units currently in inventory. Upton wants to identify the production and inventory plan for the next six months that will meet the expected demand each month while minimizing production and inventory costs.

3.12.1 DEFINING THE DECISION VARIABLES

The basic decision Upton's management team faces is how many units to manufacture in each of the next six months. We will represent these decision variables as follows:

P_1 = number of units to produce in month 1

P_2 = number of units to produce in month 2

P_3 = number of units to produce in month 3

P_4 = number of units to produce in month 4

P_5 = number of units to produce in month 5

P_6 = number of units to produce in month 6

3.12.2 DEFINING THE OBJECTIVE FUNCTION

The objective in this problem is to minimize the total production and inventory costs. The total production cost is computed easily as:

$$\text{Production Cost} = 240P_1 + 250P_2 + 265P_3 + 285P_4 + 280P_5 + 260P_6$$

The inventory cost is a bit more tricky to compute. The cost of holding a unit in inventory each month is 1.5% of the production cost in the same month. So, the unit inventory cost is \$3.60 in month 1 (i.e., 1.5% × \$240 = \$3.60), \$3.75 in month 2 (i.e., 1.5% × \$250 = \$3.75), and so on. The number of units held each month is to be computed as the average of the beginning and ending inventory for the month. Of course, the beginning inventory in any given month is equal to the ending inventory from the previous month. So if we let B_i represent the beginning inventory for month i, the total inventory cost is given by:

$$\begin{aligned}\text{Inventory Cost} = {} & 3.6(B_1 + B_2)/2 + 3.75(B_2 + B_3)/2 + 3.98(B_3 + B_4)/2 \\ & + 4.28(B_4 + B_5)/2 + 4.20(B_5 + B_6)/2 + 3.9(B_6 + B_7)/2\end{aligned}$$

Note that the first term in the previous formula computes the inventory cost for month 1 using B_1 as the beginning inventory for month 1 and B_2 as the ending inventory for month 1. Thus, the objective function for this problem is given as:

$$\text{MIN:} \quad \left.\begin{aligned} & 240P_1 + 250P_2 + 265P_3 + 285P_4 + 280P_5 + 260P_6 \\ & + 3.6(B_1 + B_2)/2 + 3.75(B_2 + B_3)/2 + 3.98(B_3 + B_4)/2 \\ & + 4.28(B_4 + B_5)/2 + 4.20(B_5 + B_6)/2 + 3.9(B_6 + B_7)/2 \end{aligned}\right\} \text{total cost}$$

3.12.3 DEFINING THE CONSTRAINTS

There are two sets of constraints that apply to this problem. First, the number of units produced each month cannot exceed the maximum production levels stated in the problem. However, we also must make sure that the number of units produced each month is no less than one half of the maximum production capacity for the month. These conditions can be expressed concisely as follows:

$$\begin{aligned}
2{,}000 \le P_1 \le 4{,}000 \quad & \} \text{ production level for month 1} \\
1{,}750 \le P_2 \le 3{,}500 \quad & \} \text{ production level for month 2} \\
2{,}000 \le P_3 \le 4{,}000 \quad & \} \text{ production level for month 3} \\
2{,}250 \le P_4 \le 4{,}500 \quad & \} \text{ production level for month 4} \\
2{,}000 \le P_5 \le 4{,}000 \quad & \} \text{ production level for month 5} \\
1{,}750 \le P_6 \le 3{,}500 \quad & \} \text{ production level for month 6}
\end{aligned}$$

These restrictions simply place the appropriate lower and upper limits on the values that each of the decision variables may assume. Similarly, we must ensure that the ending inventory each month falls between the minimum and maximum allowable inventory levels of 1,500 and 6,000, respectively. In general, the ending inventory for any month is computed as:

$$\text{Ending Inventory} = \text{Beginning Inventory} + \text{Units Produced} - \text{Units Sold}$$

Thus, the following restrictions indicate that the ending inventory in each of the next six months (after meeting the demand for the month) must fall between 1,500 and 6,000.

$$
\begin{array}{ll}
1{,}500 \le B_1 + P_1 - 1{,}000 \le 6{,}000 & \} \text{ ending inventory for month 1} \\
1{,}500 \le B_2 + P_2 - 4{,}500 \le 6{,}000 & \} \text{ ending inventory for month 2} \\
1{,}500 \le B_3 + P_3 - 6{,}000 \le 6{,}000 & \} \text{ ending inventory for month 3} \\
1{,}500 \le B_4 + P_4 - 5{,}500 \le 6{,}000 & \} \text{ ending inventory for month 4} \\
1{,}500 \le B_5 + P_5 - 3{,}500 \le 6{,}000 & \} \text{ ending inventory for month 5} \\
1{,}500 \le B_6 + P_6 - 4{,}000 \le 6{,}000 & \} \text{ ending inventory for month 6}
\end{array}
$$

Finally, to ensure that the beginning balance in one month equals the ending balance from the previous month, we have the following additional restrictions:

$$
\begin{aligned}
B_2 &= B_1 + P_1 - 1{,}000 \\
B_3 &= B_2 + P_2 - 4{,}500 \\
B_4 &= B_3 + P_3 - 6{,}000 \\
B_5 &= B_4 + P_4 - 5{,}500 \\
B_6 &= B_5 + P_5 - 3{,}500 \\
B_7 &= B_6 + P_6 - 4{,}000
\end{aligned}
$$

3.12.4 IMPLEMENTING THE MODEL

The LP problem for Upton's production and inventory planning problem may be summarized as:

$$
\text{MIN:} \quad \left.\begin{array}{l}
240P_1 + 250P_2 + 265P_3 + 285P_4 + 280P_5 + 260P_6 \\
+ 3.6(B_1 + B_2)/2 + 3.75(B_2 + B_3)/2 + 3.98(B_3 + B_4)/2 \\
+ 4.28(B_4 + B_5)/2 + 4.20(B_5 + B_6)/2 + 3.9(B_6 + B_7)/2
\end{array}\right\} \text{ total cost}
$$

Subject to:

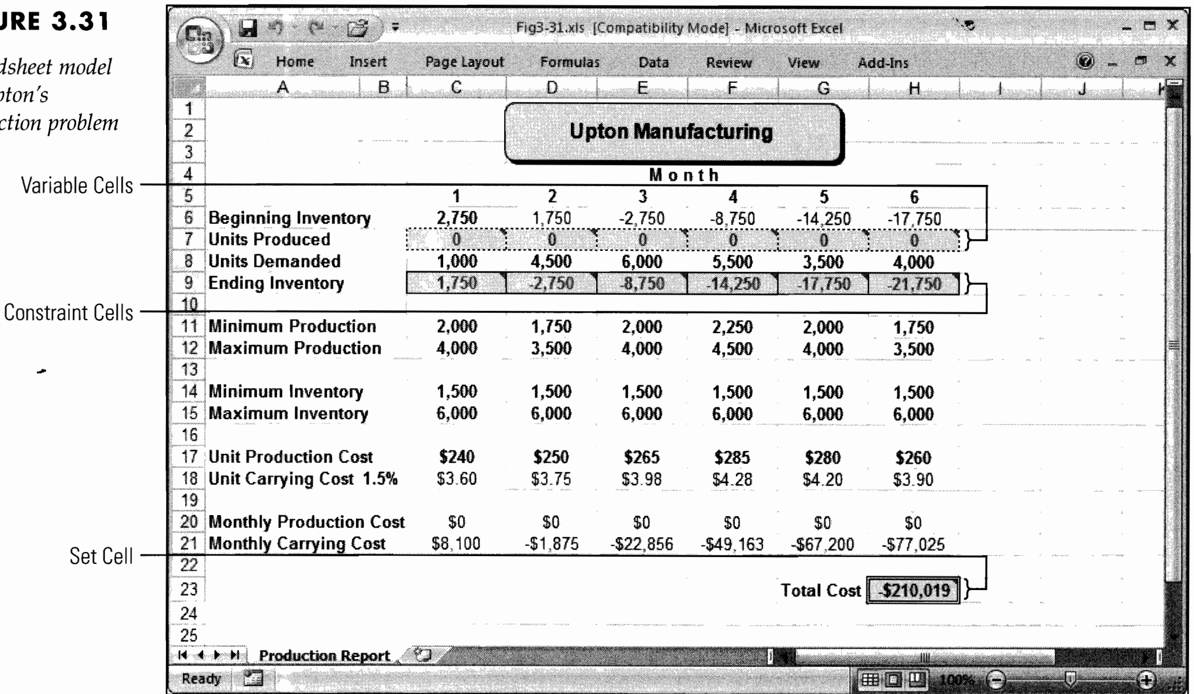
$$
\begin{array}{ll}
2{,}000 \le P_1 \le 4{,}000 & \} \text{ production level for month 1} \\
1{,}750 \le P_2 \le 3{,}500 & \} \text{ production level for month 2} \\
2{,}000 \le P_3 \le 4{,}000 & \} \text{ production level for month 3} \\
2{,}250 \le P_4 \le 4{,}500 & \} \text{ production level for month 4} \\
2{,}000 \le P_5 \le 4{,}000 & \} \text{ production level for month 5} \\
1{,}750 \le P_6 \le 3{,}500 & \} \text{ production level for month 6} \\
1{,}500 \le B_1 + P_1 - 1{,}000 \le 6{,}000 & \} \text{ ending inventory for month 1} \\
1{,}500 \le B_2 + P_2 - 4{,}500 \le 6{,}000 & \} \text{ ending inventory for month 2} \\
1{,}500 \le B_3 + P_3 - 6{,}000 \le 6{,}000 & \} \text{ ending inventory for month 3} \\
1{,}500 \le B_4 + P_4 - 5{,}500 \le 6{,}000 & \} \text{ ending inventory for month 4} \\
1{,}500 \le B_5 + P_5 - 3{,}500 \le 6{,}000 & \} \text{ ending inventory for month 5} \\
1{,}500 \le B_6 + P_6 - 4{,}000 \le 6{,}000 & \} \text{ ending inventory for month 6}
\end{array}
$$

where:

$$
\begin{aligned}
B_2 &= B_1 + P_1 - 1{,}000 \\
B_3 &= B_2 + P_2 - 4{,}500 \\
B_4 &= B_3 + P_3 - 6{,}000 \\
B_5 &= B_4 + P_4 - 5{,}500 \\
B_6 &= B_5 + P_5 - 3{,}500 \\
B_7 &= B_6 + P_6 - 4{,}000
\end{aligned}
$$

FIGURE 3.31

Spreadsheet model for Upton's production problem



	Month					
	1	2	3	4	5	6
Beginning Inventory	2,750	1,750	-2,750	-8,750	-14,250	-17,750
Units Produced	0	0	0	0	0	0
Units Demanded	1,000	4,500	6,000	5,500	3,500	4,000
Ending Inventory	1,750	-2,750	-8,750	-14,250	-17,750	-21,750
Minimum Production	2,000	1,750	2,000	2,250	2,000	1,750
Maximum Production	4,000	3,500	4,000	4,500	4,000	3,500
Minimum Inventory	1,500	1,500	1,500	1,500	1,500	1,500
Maximum Inventory	6,000	6,000	6,000	6,000	6,000	6,000
Unit Production Cost	$240	$250	$265	$285	$280	$260
Unit Carrying Cost 1.5%	$3.60	$3.75	$3.98	$4.28	$4.20	$3.90
Monthly Production Cost	$0	$0	$0	$0	$0	$0
Monthly Carrying Cost	$8,100	-$1,875	-$22,856	-$49,163	-$67,200	-$77,025
					Total Cost	-$210,019

Key Cell Formulas

Cell	Formula	Copied to
C9	=C6+C7-C8	D9:H9
D6	=C9	E6:H6
C18	=$B$18*C17	D18:H18
C20	=C17*C7	D20:H20
C21	=C18*(C6+C9)/2	D21:H21
H23	=SUM(C20:H21)	--

A convenient way of implementing this model is shown in Figure 3.31 (and file Fig3-31.xls on your data disk). Cells C7 through H7 in this spreadsheet represent the number of air compressors to produce in each month and therefore correspond to the decision variables (P_1 through P_6) in our model. We will place appropriate upper and lower bounds on these cells to enforce the restrictions represented by the first six constraints in our model. The estimated demands for each time period are listed just below the decision variables in cells C8 through H8.

With the beginning inventory level of 2,750 entered in cell C6, the ending inventory for month 1 is computed in cell C9 as follows:

Formula for cell C9: =C6+C7-C8

(Copy to cells D9 through H9.)

This formula can be copied to cells D9 through H9 to compute the ending inventory levels for each of the remaining months. We will place appropriate lower and upper limits on these cells to enforce the restrictions indicated by the second set of six constraints in our model.

To ensure that the beginning inventory in month 2 equals the ending inventory from month 1, we place the following formula in cell D6:

Formula for cell D6: =C9
(Copy to cells E6 through H6.)

This formula can be copied to cells E6 through H6 to ensure that the beginning inventory levels in each month equal the ending inventory levels from the previous month. It is important to note that because the beginning inventory levels can be calculated directly from the ending inventory levels, there is no need to specify these cells as constraint cells to Solver.

With the monthly unit production costs entered in cell C17 through H17, the monthly unit carrying costs are computed in cells C18 through H18 as follows:

Formula for cell C18: =$B$18*C17
(Copy to cells D18 through H18.)

The total monthly production and inventory costs are then computed in rows 20 and 21 as follows:

Formula for cell C20: =C17*C7
(Copy to cells D20 through H20.)

Formula for cell C21: =C18*(C6 + C9)/2
(Copy to cells D21 through H21.)

Finally, the objective function representing the total production and inventory costs for the problem is implemented in cell H23 as follows:

Formula for cell H23: =SUM(C20:H21)

3.12.5 SOLVING THE MODEL

Figure 3.32 shows the Solver parameters required to solve this problem. The optimal solution is shown in Figure 3.33.

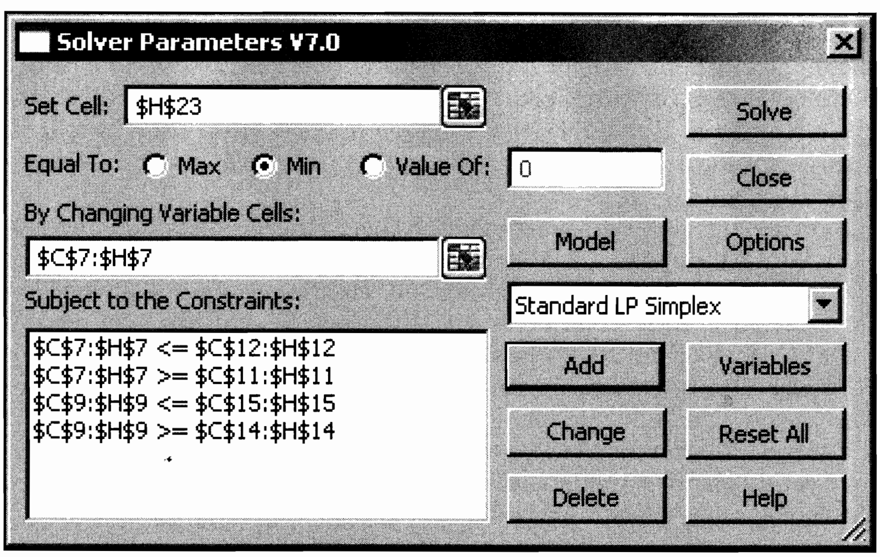


FIGURE 3.32

Solver parameters for the production problem

FIGURE 3.33

Optimal solution to Upton's production problem

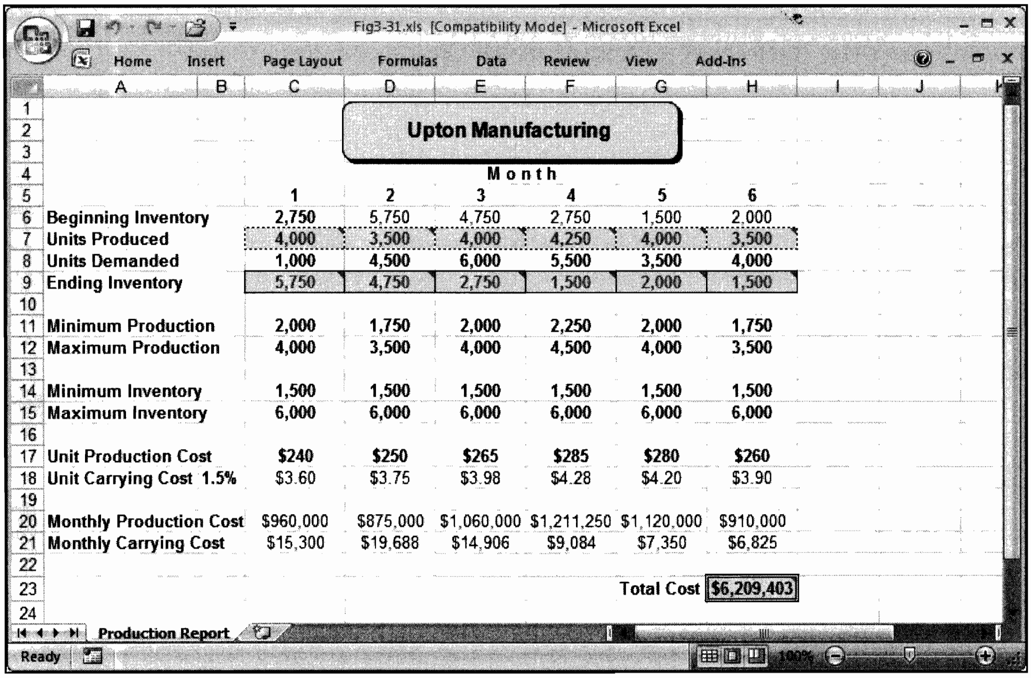


Fig3-31.xls [Compatibility Mode] - Microsoft Excel

Upton Manufacturing

	Month					
	1	2	3	4	5	6
Beginning Inventory	2,750	5,750	4,750	2,750	1,500	2,000
Units Produced	4,000	3,500	4,000	4,250	4,000	3,500
Units Demanded	1,000	4,500	6,000	5,500	3,500	4,000
Ending Inventory	5,750	4,750	2,750	1,500	2,000	1,500
Minimum Production	2,000	1,750	2,000	2,250	2,000	1,750
Maximum Production	4,000	3,500	4,000	4,500	4,000	3,500
Minimum Inventory	1,500	1,500	1,500	1,500	1,500	1,500
Maximum Inventory	6,000	6,000	6,000	6,000	6,000	6,000
Unit Production Cost	$240	$250	$265	$285	$280	$260
Unit Carrying Cost 1.5%	$3.60	$3.75	$3.98	$4.28	$4.20	$3.90
Monthly Production Cost	$960,000	$875,000	$1,060,000	$1,211,250	$1,120,000	$910,000
Monthly Carrying Cost	$15,300	$19,688	$14,906	$9,084	$7,350	$6,825
					Total Cost	$6,209,403

Production Report

3.12.6 ANALYZING THE SOLUTION

The optimal solution shown in Figure 3.33 indicates that Upton should produce 4,000 units in period 1, 3,500 units in period 2, 4,000 units in period 3, 4,250 units in period 4, 4,000 units in period 5, and 3,500 units in period 6. Although the demand for air compressors in month 1 can be met by the beginning inventory, production in month 1 is required to build inventory for future months in which demand exceeds the available production capacity. Notice that this production schedule calls for the company to operate at full production capacity in all months except month 4. Month 4 is expected to have the highest per unit production cost. Therefore, it is more economical to produce extra units in prior months and hold them in inventory for sale in month 4.

It is important to note that although the solution to this problem provides a production plan for the next six months, it does not bind Upton's management team to implement this particular solution throughout the next six months. At an operational level, the management team is most concerned with the decision that must be made now—namely, the number of units to schedule for production in month 1. At the end of month 1, Upton's management should update the inventory, demand, and cost estimates, and re-solve the model to identify the production plan for the next six months (presently months 2 through 7). At the end of month 2, this process should be repeated. Thus, multiperiod planning models such as this should be used repeatedly on a periodic basis as part of a rolling planning process.

3.13 A Multi-Period Cash Flow Problem

Numerous business problems involve decisions that have a ripple effect on future decisions. In the previous example, we saw how the manufacturing plans for one time period can affect the amount of resources available and the inventory carried in subsequent time periods. Similarly, many financial decisions involve multiple time periods because the amount of money invested or spent at one point in time directly affects the amount of money available in subsequent time periods. In these types of multi-period problems, it can be difficult to account for the consequences of a current decision on future time periods without an LP model. The formulation of such a model is illustrated next in an example from the world of finance.

Taco-Viva is a small but growing restaurant chain specializing in Mexican fast food. The management of the company has decided to build a new location in Wilmington, North Carolina, and wants to establish a construction fund (or sinking fund) to pay for the new facility. Construction of the restaurant is expected to take six months and cost $800,000. Taco-Viva's contract with the construction company requires it to make payments of $250,000 at the end of the second and fourth months, and a final payment of $300,000 at the end of the sixth month when the restaurant is completed. The company can use four investment opportunities to establish the construction fund; these investments are summarized in the following table:

Investment	Available in Month	Months to Maturity	Yield at Maturity
A	1, 2, 3, 4, 5, 6	1	1.8%
B	1, 3, 5	2	3.5%
C	1, 4	3	5.8%
D	1	6	11.0%

The table indicates that investment A will be available at the beginning of each of the next six months, and funds invested in this manner mature in one month with a yield of 1.8%. Funds can be placed in investment C only at the beginning of months 1 and/or 4, and mature at the end of three months with a yield of 5.8%.

The management of Taco-Viva needs to determine the investment plan that allows them to meet the required schedule of payments while placing the least amount of money in the construction fund.

This is a multi-period problem because a six-month planning horizon must be considered. That is, Taco-Viva must plan which investment alternatives to use at various times during the next six months.

3.13.1 DEFINING THE DECISION VARIABLES

The basic decision faced by the management of Taco-Viva is how much money to place in each investment vehicle during each time period when the investment opportunities are available. To model this problem, we need different variables to represent each investment/time period combination. This can be done as:

$A_1, A_2, A_3, A_4, A_5, A_6 =$ the amount of money (in \$1,000s) placed in investment A at the beginning of months 1, 2, 3, 4, 5, and 6, respectively

$B_1, B_3, B_5 =$ the amount of money (in \$1,000s) placed in investment B at the beginning of months 1, 3, and 5, respectively

$C_1, C_4 =$ the amount of money (in \$1,000s) placed in investment C at the beginning of months 1 and 4, respectively

$D_1 =$ the amount of money (in \$1,000s) placed in investment D at the beginning of month 1

Notice that all variables are expressed in units of thousands of dollars to maintain a reasonable scale for this problem. So, keep in mind that when referring to the amount of money represented by our variables, we mean the amount in thousands of dollars.

3.13.2 DEFINING THE OBJECTIVE FUNCTION

Taco-Viva's management wants to minimize the amount of money it must place in the construction fund initially to cover the payments that will be due under the contract. At the beginning of month 1, the company wants to invest some amount of money that, along with its investment earnings, will cover the required payments without an additional infusion of cash from the company. Because A_1, B_1, C_1, and D_1 represent the initial amounts invested by the company in month 1, the objective function for the problem is:

$$\text{MIN:} \quad A_1 + B_1 + C_1 + D_1 \quad \text{\} total cash invested at the beginning of month 1}$$

3.13.3 DEFINING THE CONSTRAINTS

To formulate the cash-flow constraints for this problem, it is important to clearly identify: (1) when the different investments can be made, (2) when the different investments will mature, and (3) how much money will be available when each investment matures. Figure 3.34 summarizes this information.

The negative values, represented by −1 in Figure 3.34, indicate when dollars can flow *into* each investment. The positive values indicate how much these same dollars will be worth when the investment matures, or when dollars flow *out* of each investment. The double-headed arrow symbols indicate time periods in which funds remain in a particular investment. For example, the third row of the table in Figure 3.34 indicates that every dollar placed in investment C at the beginning of month 1 will be worth \$1.058 when this investment matures three months later—at the *beginning* of month 4. (Note that the beginning of month 4 occurs at virtually the same instant as the *end* of month 3. Thus, there is no practical difference between the beginning of one time period and the end of the previous time period.)

Assuming that the company invests the amounts represented by A_1, B_1, C_1, and D_1 at the beginning of month 1, how much money would be available to reinvest or make the required payments at the beginning of months 2, 3, 4, 5, 6, and 7? The answer to this question allows us to generate the set of cash-flow constraints needed for this problem.

As indicated by the second column of Figure 3.34, the only funds maturing at the beginning of month 2 are those placed in investment A at the beginning of month 1 (A_1). The value of the funds maturing at the beginning of month 2 is $\$1.018A_1$. Because no payments are required at the beginning of month 2, all the maturing funds

Investment	1	2	3	4	5	6	7
	Cash Inflow/Outflow at the Beginning of Month						
A_1	−1	1.018					
B_1	−1	↔	1.035				
C_1	−1	↔	↔	1.058			
D_1	−1	↔	↔	↔	↔	↔	1.11
A_2		−1	1.018				
A_3			−1	1.018			
B_3			−1	↔	1.035		
A_4				−1	1.018		
C_4				−1	↔	↔	1.058
A_5					−1	1.018	
B_5					−1	↔	1.035
A_6						−1	1.018
Req'd Payments (in \$1,000s)	\$0	\$0	\$250	\$0	\$250	\$0	\$300

FIGURE 3.34

Cash-flow summary table for Taco-Viva's investment opportunities

must be reinvested. But the only new investment opportunity available at the beginning of month 2 is investment A (A_2). Thus, the amount of money placed in investment A at the beginning of month 2 must be $\$1.018A_1$. This is expressed by the constraint:

$$1.018A_1 = A_2 + 0 \qquad \} \text{ cash flow for month 2}$$

This constraint indicates that the total amount of money maturing at the beginning of month 2 ($1.018A_1$) must equal the amount of money reinvested at the beginning of month 2 (A_2) plus any payment due in month 2 (\$0).

Now, consider the cash flows that will occur during month 3. At the beginning of month 3, any funds that were placed in investment B at the beginning of month 1 (B_1) will mature and be worth a total of $\$1.035B_1$. Similarly, any funds placed in investment A at the beginning of month 2 (A_2) will mature and be worth a total of $\$1.018A_2$. Because a payment of \$250,000 is due at the beginning of month 3, we must ensure that the funds maturing at the beginning of month 3 are sufficient to cover this payment, and that any remaining funds are placed in the investment opportunities available at the beginning of month 3 (A_3 and B_3). This requirement can be stated algebraically as:

$$1.035B_1 + 1.018A_2 = A_3 + B_3 + 250 \qquad \} \text{ cash flow for month 3}$$

This constraint indicates that the total amount of money maturing at the beginning of month 3 ($1.035B_1 + 1.018A_2$) must equal the amount of money reinvested at the beginning of month 3 ($A_3 + B_3$) plus the payment due at the beginning of month 3 (\$250,000).

The same logic we applied to generate the cash-flow constraints for months 2 and 3 also can be used to generate cash-flow constraints for the remaining months. Doing so produces a cash-flow constraint for each month that takes on the general form:

$$\begin{pmatrix} \text{Total \$ amount} \\ \text{maturing at the} \\ \text{beginning} \\ \text{of the month} \end{pmatrix} = \begin{pmatrix} \text{Total \$ amount} \\ \text{reinvested at the} \\ \text{beginning} \\ \text{of the month} \end{pmatrix} + \begin{pmatrix} \text{Payment} \\ \text{due at the} \\ \text{beginning} \\ \text{of the month} \end{pmatrix}$$

Using this general definition of the cash flow relationships, the constraints for the remaining months are represented by:

$$1.058C_1 + 1.018A_3 = A_4 + C_4 \quad \} \text{ cash flow for month 4}$$
$$1.035B_3 + 1.018A_4 = A_5 + B_5 + 250 \quad \} \text{ cash flow for month 5}$$
$$1.018A_5 = A_6 \quad \} \text{ cash flow for month 6}$$
$$1.11D_1 + 1.058C_4 + 1.035B_5 + 1.018A_6 = 300 \quad \} \text{ cash flow for month 7}$$

To implement these constraints in the spreadsheet, we must express them in a slightly different (but algebraically equivalent) manner. Specifically, to conform to our general definition of an equality constraint ($f(X_1, X_2, \ldots, X_n) = b$) we need to rewrite the cash-flow constraints so that all the *variables* in each constraint appear on the LHS of the equal sign, and a numeric constant appears on the RHS of the equal sign. This can be done as:

$$1.018A_1 - 1A_2 = 0 \quad \} \text{ cash flow for month 2}$$
$$1.035B_1 + 1.018A_2 - 1A_3 - 1B_3 = 250 \quad \} \text{ cash flow for month 3}$$
$$1.058C_1 + 1.018A_3 - 1A_4 - 1C_4 = 0 \quad \} \text{ cash flow for month 4}$$
$$1.035B_3 + 1.018A_4 - 1A_5 - 1B_5 = 250 \quad \} \text{ cash flow for month 5}$$
$$1.018A_5 - 1A_6 = 0 \quad \} \text{ cash flow for month 6}$$
$$1.11D_1 + 1.058C_4 + 1.035B_5 + 1.018A_6 = 300 \quad \} \text{ cash flow for month 7}$$

There are two important points to note about this alternate expression of the constraints. First, each constraint takes on the following general form, which is algebraically equivalent to our previous general definition for the cash-flow constraints:

$$\begin{pmatrix}\text{Total \$ amount}\\ \text{maturing at the}\\ \text{beginning}\\ \text{of the month}\end{pmatrix} - \begin{pmatrix}\text{Total \$ amount}\\ \text{reinvested at the}\\ \text{beginning}\\ \text{of the month}\end{pmatrix} = \begin{pmatrix}\text{Payment}\\ \text{due at the}\\ \text{beginning}\\ \text{of the month}\end{pmatrix}$$

Although the constraints look slightly different in this form, they enforce the same relationships among the variables as expressed by the earlier constraints.

Second, the LHS coefficients in the alternate expression of the constraints correspond directly to the values listed in the cash-flow summary table in Figure 3.34. That is, the coefficients in the constraint for month 2 correspond to the values in the column for month 2 in Figure 3.34; the coefficients for month 3 correspond to the values in the column for month 3, and so on. This relationship is true for all the constraints and will be very helpful in implementing this model in the spreadsheet.

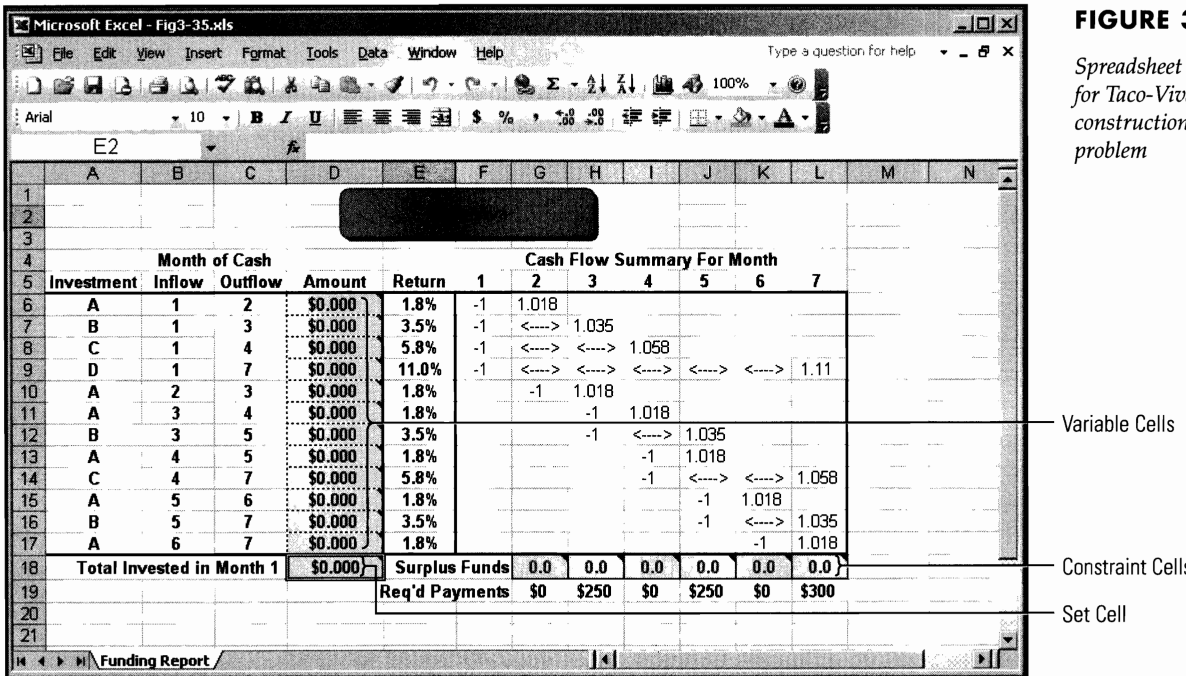
3.13.4 IMPLEMENTING THE MODEL

The LP model for Taco-Viva's construction fund problem is summarized as:

MIN: $A_1 + B_1 + C_1 + D_1$ } cash invested at beginning of month 1

Subject to:

$$\begin{aligned}
&1.018A_1 - 1A_2 &&= 0 &&\} \text{ cash flow for month 2}\\
&1.035B_1 + 1.018A_2 - 1A_3 - 1B_3 &&= 250 &&\} \text{ cash flow for month 3}\\
&1.058C_1 + 1.018A_3 - 1A_4 - 1C_4 &&= 0 &&\} \text{ cash flow for month 4}\\
&1.035B_3 + 1.018A_4 - 1A_5 - 1B_5 &&= 250 &&\} \text{ cash flow for month 5}\\
&1.018A_5 - 1A_6 &&= 0 &&\} \text{ cash flow for month 6}\\
&1.11D_1 + 1.058C_4 + 1.035B_5 + 1.018A_6 &&= 300 &&\} \text{ cash flow for month 7}\\
&A_i, B_i, C_i, D_i, \geq 0, \text{ for all } i && &&\} \text{ nonnegativity conditions}
\end{aligned}$$



	A	B	C	D	E	F	G	H	I	J	K	L
4		Month of Cash				Cash Flow Summary For Month						
5	Investment	Inflow	Outflow	Amount	Return	1	2	3	4	5	6	7
6	A	1	2	$0.000	1.8%	-1	1.018					
7	B	1	3	$0.000	3.5%	-1	<---->	1.035				
8	C	1	4	$0.000	5.8%	-1	<---->	<---->	1.058			
9	D	1	7	$0.000	11.0%	-1	<---->	<---->	<---->	<---->	<---->	1.11
10	A	2	3	$0.000	1.8%		-1	1.018				
11	A	3	4	$0.000	1.8%			-1	1.018			
12	B	3	5	$0.000	3.5%			-1	<---->	1.035		
13	A	4	5	$0.000	1.8%				-1	1.018		
14	C	4	7	$0.000	5.8%				-1	<---->	<---->	1.058
15	A	5	6	$0.000	1.8%					-1	1.018	
16	B	5	7	$0.000	3.5%					-1	<---->	1.035
17	A	6	7	$0.000	1.8%						-1	1.018
18	Total Invested in Month 1			$0.000	Surplus Funds		0.0	0.0	0.0	0.0	0.0	0.0
19					Req'd Payments		$0	$250	$0	$250	$0	$300

Key Cell Formulas

Cell	Formula	Copied to
D18	=SUMIF(B6:B17,1,D6:D17)	--
F6	=IF($B6=F$5,-1,IF($C6=F$5,1+$E6,IF(AND($B6<F$5,$C6>F$5),"<---->","")))	F6:L17
G18	=SUMPRODUCT(G6:G17,$D$6:$D$17)	H18:L18

FIGURE 3.35

Spreadsheet model for Taco-Viva's construction fund problem

One approach to implementing this model is shown in Figure 3.35 (and file Fig3-35.xls on your data disk). The first three columns of this spreadsheet summarize the different investment options that are available and the months in which money may flow into and out of these investments. Cells D6 through D17 represent the decision variables in our model and indicate the amount of money (in $1,000s) to be placed in each of the possible investments.

The objective function for this problem requires that we compute the total amount of money being invested in month 1. This was done in cell D18 as follows:

Formula for cell D18: =SUMIF(B6:B17,1,D6:D17)

This SUMIF function compares the values in cells B6 through B17 to the value 1 (its second argument). If any of the values in B6 through B17 equal 1, it sums the corresponding values in cells D6 through D17. In this case, the values in cells B6 through B9 all equal 1; therefore, the function returns the sum of the values in cells D6 through D9. Note that although we could have implemented the objective using the formula SUM(D6:D9), the previous SUMIF formula makes for a more modifiable and reliable model. If any of the values in column B are changed to or from 1, the SUMIF function continues to represent the appropriate objective function, whereas the SUM function would not.

Our next job is to implement the cash inflow/outflow table described earlier in Figure 3.34. Recall that each row in Figure 3.34 corresponds to the cash flows associated with a particular investment alternative. This table can be implemented in our spreadsheet using the following formula:

Formula for cell F6: =IF($B6=F$5,-1,IF($C6=F$5,1+$E6,IF(AND($B6<F$5,$C6>F$5)," <--->","")))
(Copy to cells F6 through L17.)

This formula first checks to see if the "month of cash inflow" value in column B matches the month indicator value in row 5. If so, the formula returns the value −1. Otherwise, it goes on to check to see if the "month of cash outflow" value in column C matches the month indicator value in row 5. If so, the formula returns a value equal to 1 plus the return for the investment (from column E). If neither of the first two conditions are met, the formula next checks whether the current month indicator in row 5 is larger than the "month of cash inflow" value (column B) and smaller than the "month of cash outflow" value (column C). If so, the formula returns the characters "<--->" to indicate periods in which funds neither flow into or out of a particular investment. Finally, if none of the previous three conditions are met, the formula simply returns an empty (or null) string "". Although this formula looks a bit intimidating, it is simply a set of three nested IF functions. More important, it automatically updates the cash flow summary if any of the values in columns B, C, or E are changed, increasing the reliability and modifiability of the model.

Earlier, we noted that the values listed in columns 2 through 7 of the cash inflow/outflow table correspond directly to the coefficients appearing in the various cash-flow constraints. This property allows us to implement the cash-flow constraints in the spreadsheet conveniently. For example, the LHS formula for the cash-flow constraint for month 2 is implemented in cell G18 through the formula:

Formula in cell G18: =SUMPRODUCT(G6:G17,$D$6:$D$17)
(Copy to H18 through L18.)

This formula multiplies each entry in the range G6 through G17 by the corresponding entry in the range D6 through D17 and then sums these individual products. This formula is copied to cells H18 through L18. (Notice that the SUMPRODUCT( ) formula treats cells containing labels and null strings as if they contained the value zero.) Take a moment now to verify that the formulas in cells G18 through L18 correspond to the LHS formulas of the cash-flow constraints in our model. Cells G19 through L19 list the RHS values for the cash-flow constraints.

3.13.5 SOLVING THE MODEL

To find the optimal solution to this model, we must indicate to Solver the set cell, variable cells, and constraint cells identified in Figure 3.35. Figure 3.36 shows the Solver parameters required to solve this model. The optimal solution is shown in Figure 3.37.

3.13.6 ANALYZING THE SOLUTION

The value of the set cell (D18) in Figure 3.37 indicates that a total of $741,363 must be invested to meet the payments on Taco-Viva's construction project. Cells D6 and D8 indicate that approximately $241,237 should be placed in investment A at the beginning of month 1 ($A_1 = 241.237$) and approximately $500,126 should be placed in investment C ($C_1 = 500.126$).

At the beginning of month 2, the funds placed in investment A at the beginning of month 1 will mature and will be worth $245,580 ($241,237 \times 1.018 = 245,580$). The value

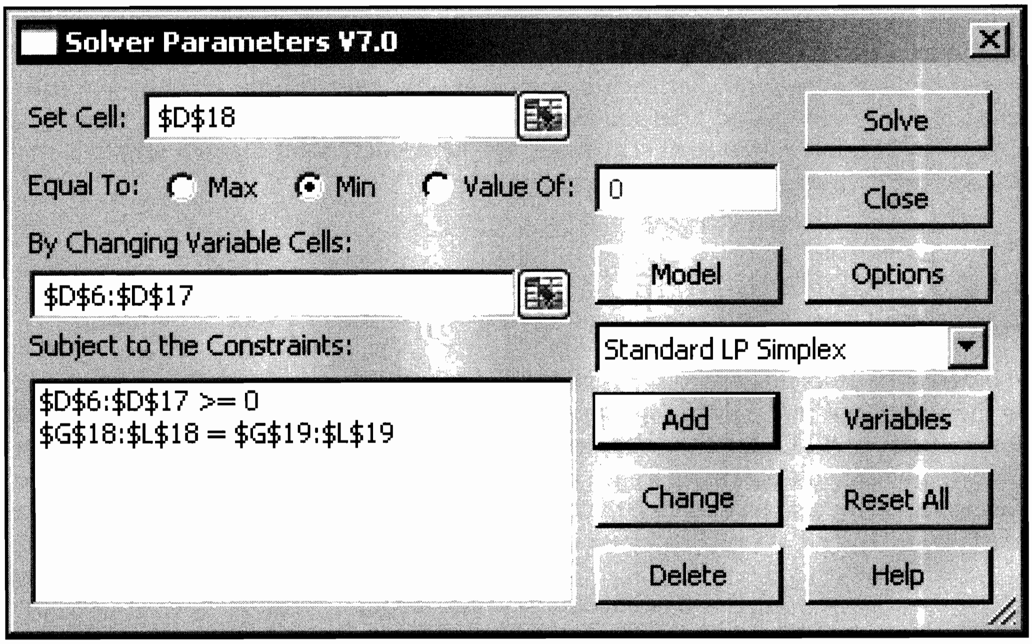
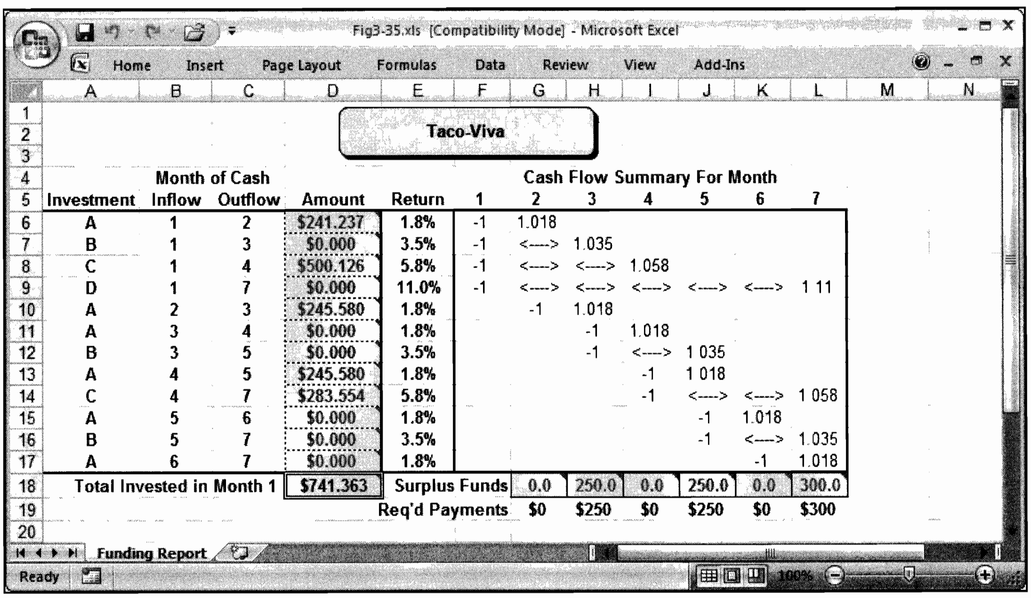


FIGURE 3.36

Solver parameters for the construction fund problem



	Month of Cash				Cash Flow Summary For Month						
Investment	Inflow	Outflow	Amount	Return	1	2	3	4	5	6	7
A	1	2	$241.237	1.8%	-1	1.018					
B	1	3	$0.000	3.5%	-1	<---->	1.035				
C	1	4	$500.126	5.8%	-1	<---->	<---->	1.058			
D	1	7	$0.000	11.0%	-1	<---->	<---->	<---->	<---->	<---->	1.11
A	2	3	$245.580	1.8%		-1	1.018				
A	3	4	$0.000	1.8%			-1	1.018			
B	3	5	$0.000	3.5%			-1	<---->	1.035		
A	4	5	$245.580	1.8%				-1	1.018		
C	4	7	$283.554	5.8%				-1	<---->	<---->	1.058
A	5	6	$0.000	1.8%					-1	1.018	
B	5	7	$0.000	3.5%					-1	<---->	1.035
A	6	7	$0.000	1.8%						-1	1.018
Total Invested in Month 1			$741.363	Surplus Funds		0.0	250.0	0.0	250.0	0.0	300.0
				Req'd Payments		$0	$250	$0	$250	$0	$300

FIGURE 3.37

Optimal solution to Taco-Viva's construction fund problem

in cell D10 indicates that these funds should be placed back into investment A at the beginning of month 2 ($A_2 = 245.580$).

At the beginning of month 3, the first $250,000 payment is due. At that time, the funds placed in investment A at the beginning of month 2 will mature and will be worth $250,000 ($1.018 \times 245{,}580 = 250{,}000$) – allowing us to make this payment.

At the beginning of month 4, the funds placed in investment C at the beginning of month 1 will mature and will be worth $529,134. Our solution indicates that $245,580 of this amount should be placed in investment A ($A_4 = 245.580$) and that the rest should be reinvested in investment C ($C_4 = 283.554$).

If you trace through the cash flows for the remaining months, you will discover that our model is doing exactly what it was designed to do. The amount of money scheduled to mature at the beginning of each month is exactly equal to the amount of money scheduled to be reinvested after required payments are made. Thus, out of an infinite number of possible investment schedules, our LP model found the one schedule that requires the least amount of money up front.

3.13.7 MODIFYING THE TACO-VIVA PROBLEM TO ACCOUNT FOR RISK (OPTIONAL)

In investment problems like this, it is not uncommon for decision makers to place limits on the amount of risk they are willing to assume. For instance, suppose that the chief financial officer (CFO) for Taco-Viva assigned the following risk ratings to each of the possible investments on a scale from 1 to 10 (where 1 represents the least risk and 10 the greatest risk). We also will assume that the CFO wants to determine an investment plan where the weighted average risk level does not exceed 5.

Investment	Risk Rating
A	1
B	3
C	8
D	6

We will need to formulate an additional constraint for each time period to ensure that the weighted average risk level never exceeds 5. To see how this can be done, let's start with month 1.

In month 1, funds can be invested in A_1, B_1, C_1, and/or D_1, and each investment is associated with a different degree of risk. To calculate the weighted average risk during month 1, we must multiply the risk factors for each investment by the proportion of money in that investment. This is represented by:

$$\text{Weighted average risk in month 1} = \frac{1A_1 + 3B_1 + 8C_1 + 6D_1}{A_1 + B_1 + C_1 + D_1}$$

We can ensure that the weighted average risk in month 1 does not exceed the value 5 by including the following constraint in our LP model:

$$\frac{1A_1 + 3B_1 + 8C_1 + 6D_1}{A_1 + B_1 + C_1 + D_1} \leq 5 \quad \} \text{ risk constraint for month 1}$$

Now, consider month 2. According to the column for month 2 in our cash inflow/outflow table, the company can have funds invested in B_1, C_1, D_1, and/or A_2 during this month. Thus, the weighted average risk that occurs in month 2 is defined by:

$$\text{Weighted average risk in month 2} = \frac{3B_1 + 8C_1 + 6D_1 + 1A_2}{B_1 + C_1 + D_1 + A_2}$$

Again, the following constraint ensures that this quantity never exceeds 5:

$$\frac{3B_1 + 8C_1 + 6D_1 + 1A_2}{B_1 + C_1 + D_1 + A_2} \leq 5 \quad \} \text{ risk constraint for month 2}$$

The risk constraints for months 3 through 6 are generated in a similar manner, and appear as:

$$\frac{8C_1 + 6D_1 + 1A_3 + 3B_3}{C_1 + D_1 + A_3 + B_3} \leq 5 \quad \} \text{ risk constraint for month 3}$$

$$\frac{6D_1 + 3B_3 + 1A_4 + 8C_4}{D_1 + B_3 + A_4 + C_4} \leq 5 \quad \} \text{ risk constraint for month 4}$$

$$\frac{6D_1 + 8C_4 + 1A_5 + 3B_5}{D_1 + C_4 + A_5 + B_5} \leq 5 \quad \} \text{ risk constraint for month 5}$$

$$\frac{6D_1 + 8C_4 + 3B_5 + 1A_6}{D_1 + C_4 + B_5 + A_6} \leq 5 \quad \} \text{ risk constraint for month 6}$$

Although the risk constraints listed here have a very clear meaning, it is easier to implement these constraints in the spreadsheet if we state them in a different (but algebraically equivalent) manner. In particular, it is helpful to eliminate the fractions on the LHS of the inequalities by multiplying each constraint through by its denominator and re-collecting the variables on the LHS of the inequality. The following steps show how to rewrite the risk constraint for month 1:

1. Multiply both sides of the inequality by the denominator:

$$(A_1 + B_1 + C_1 + D_1)\,\frac{1A_1 + 3B_1 + 8C_1 + 6D_1}{A_1 + B_1 + C_1 + D_1} \leq (A_1 + B_1 + C_1 + D_1)5$$

   to obtain:

$$1A_1 + 3B_1 + 8C_1 + 6D_1 \leq 5A_1 + 5B_1 + 5C_1 + 5D_1$$

2. Re-collect the variables on the LHS of the inequality sign:

$$(1 - 5)A_1 + (3 - 5)B_1 + (8 - 5)C_1 + (6 - 5)D_1 \leq 0$$

   to obtain:

$$-4A_1 - 2B_1 + 3C_1 + 1D_1 \leq 0$$

Thus, the following two constraints are algebraically equivalent:

$$\frac{1A_1 + 3B_1 + 8C_1 + 6D_1}{A_1 + B_1 + C_1 + D_1} \leq 5 \quad \} \text{ risk constraint for month 1}$$

$$-4A_1 - 2B_1 + 3C_1 + 1D_1 \leq 0 \quad \} \text{ risk constraint for month 1}$$

The set of values for A_1, B_1, C_1, and D_1 that satisfies the first of these constraints also satisfies the second constraint (that is, these constraints have exactly the same set of feasible values). So, it does not matter which of these constraints we use to find the optimal solution to the problem.

The remaining risk constraints are simplified in the same way, producing the following constraints:

$$\begin{array}{rcrcrcrcl}
-2B_1 & + & 3C_1 & + & 1D_1 & - & 4A_2 & \leq & 0 \quad \} \text{ risk constraint for month 2} \\
3C_1 & + & 1D_1 & - & 4A_3 & - & 2B_3 & \leq & 0 \quad \} \text{ risk constraint for month 3} \\
1D_1 & - & 2B_3 & - & 4A_4 & + & 3C_4 & \leq & 0 \quad \} \text{ risk constraint for month 4} \\
1D_1 & + & 3C_4 & - & 4A_5 & - & 2B_5 & \leq & 0 \quad \} \text{ risk constraint for month 5} \\
1D_1 & + & 3C_4 & - & 2B_5 & - & 4A_6 & \leq & 0 \quad \} \text{ risk constraint for month 6}
\end{array}$$

Notice that the coefficient for each variable in these constraints is simply the risk factor for the particular investment minus the maximum allowable weighted average risk value of 5. That is, all A_i variables have coefficients of $1 - 5 = -4$; all B_i variables have coefficients of $3 - 5 = -2$; all C_i variables have coefficients of $8 - 5 = 3$; and all D_i variables have coefficients of $6 - 5 = 1$. This observation will help us implement these constraints efficiently.

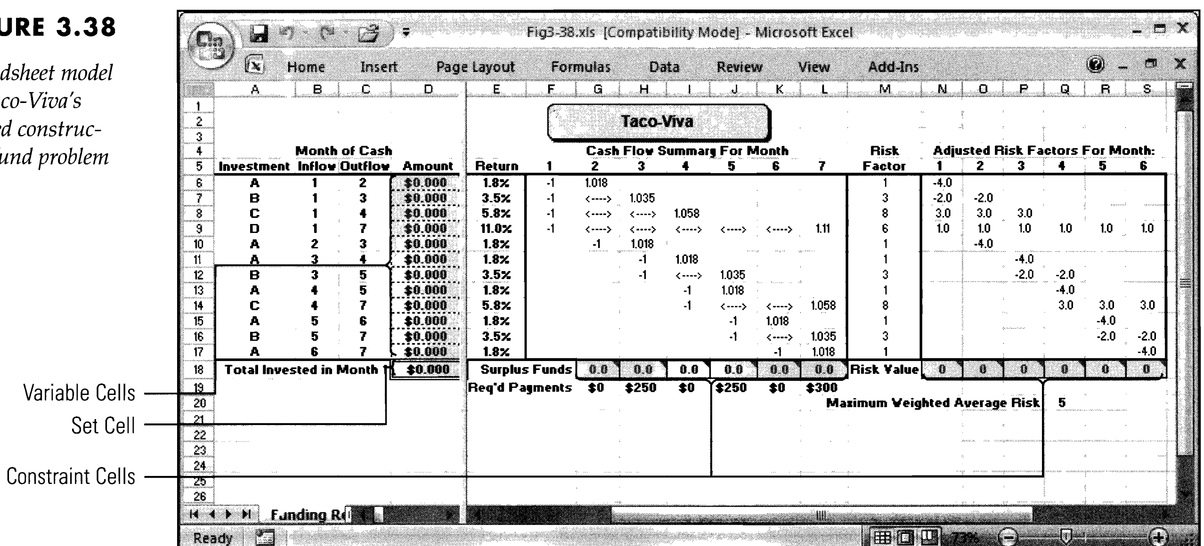
3.13.8 IMPLEMENTING THE RISK CONSTRAINTS

Figure 3.38 (and file Fig3-38.xls on your data disk) shows a split screen that illustrates an easy way to implement the risk constraints for this model. Earlier we noted that the coefficient for each variable in each risk constraint is simply the risk factor for the particular investment minus the maximum allowable weighted average risk value. Thus, the strategy in Figure 3.38 is to generate these values in the appropriate columns and rows of the spreadsheet so that the SUMPRODUCT( ) function can implement the LHS formulas for the risk constraints.

Recall that the risk constraint for each month involves only the variables representing investments that actually held funds during that month. For any given month, the investments that actually held funds during that month have the value −1 or contain a text entry starting with the "<" symbol (the first character of the "<---->" entries) in the

FIGURE 3.38

Spreadsheet model for Taco-Viva's revised construction fund problem



Key Cell Formulas

Cell	Formula	Copied to
D18	=SUMIF(B6:B17,1,D6:D17)	--
F6	=IF($B6=F$5,-1,IF($C6=F$5,1+$E6,IF(AND($B6<F$5,$C6>F$5),"<---->","")))	F6:L17
G18	=SUMPRODUCT(G6:G17,$D$6:$D$17)	H18:L18 and N18:S18
N6	=IF(OR(F6=-1,LEFT(F6)="<"),$M6-$Q$20,"")	N6:S17

corresponding column of the cash inflow/outflow summary table. For example, during month 2, funds can be invested in B_1, C_1, D_1, and/or A_2. The corresponding cells for month 2 in Figure 3.38 (cells G7, G8, G9, and G10, respectively) each contain either the value −1 or a text entry starting with the "<" symbol. Therefore, to generate the appropriate coefficients for the risk constraints, we can instruct the spreadsheet to scan the cash inflow/outflow summary for cells containing the value −1 or text entries starting with the "<" symbol, and return the correct risk constraint coefficients in the appropriate cells. To do this we enter the following formula in cell N6:

Formula in cell N6: =IF(OR(F6=-1,LEFT(F6)="<"),$M6-$Q$20,"")
(Copy to N6 through S17.)

To generate the appropriate value in cell N6, the previous formula checks if cell F6 is equal to −1 or contains a text entry that starts with the "<" symbol. If either of these conditions is true, the function takes the risk factor for the investment from cell M6 and subtracts the maximum allowable risk factor found in cell Q20; otherwise, the function returns a null string (with a value of zero). This formula is copied to the remaining cells in the range N6 through S17, as shown in Figure 3.38.

The values in cells N6 through S17 in Figure 3.38 correspond to the coefficients in the LHS formulas for each of the risk constraints formulated earlier. Thus, the LHS formula for the risk constraint for month 1 is implemented in cell N18 as:

Formula in cell N18: =SUMPRODUCT(N6:N17,$D$6:$D$17)
(Copy to O18 through S18.)

The LHS formulas for the remaining risk constraints are implemented by copying this formula to cells O18 through S18. We will tell Solver that these constraint cells must be less than or equal to zero.

3.13.9 SOLVING THE MODEL

To find the optimal solution to this model, we must communicate the appropriate information about the new risk constraints to Solver. Figure 3.39 shows the Solver parameters required to solve this model. The optimal solution is shown in Figure 3.40.

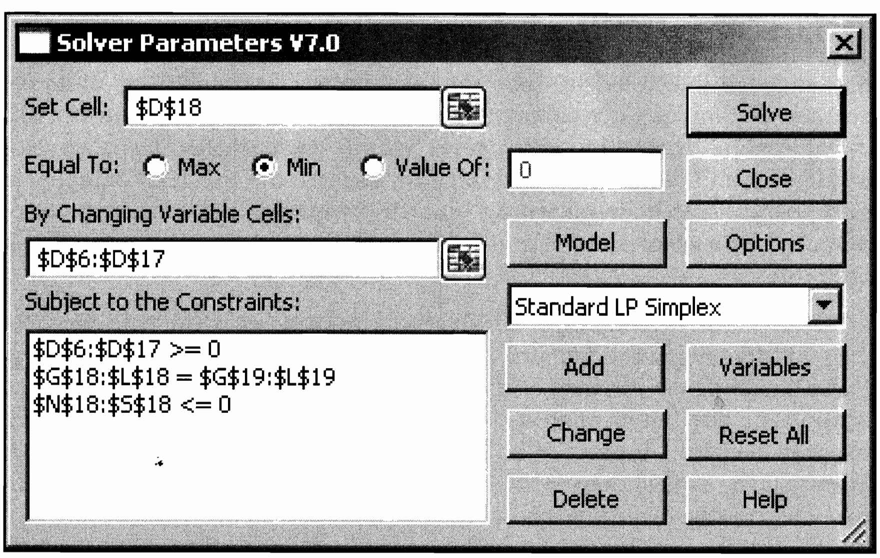


FIGURE 3.39

Solver parameters for the revised construction fund problem

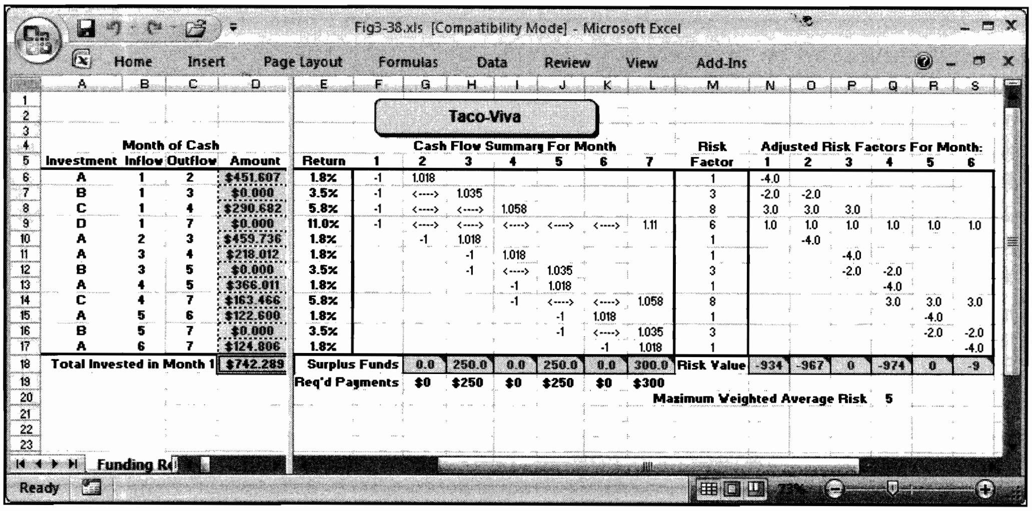


FIGURE 3.40

Optimal solution to Taco-Viva's revised construction fund problem

3.13.10 ANALYZING THE SOLUTION

The optimal solution to the revised Taco-Viva problem with risk constraints is quite different than the solution obtained earlier. In particular, the new solution requires that funds be placed in investment A in every time period. This is not too surprising given that investment A has the lowest risk rating. It may be somewhat surprising that of the remaining investments, B and D never are used. Although these investments have lower risk ratings than investment C, the combination of funds placed in investment A and C allows for the least amount of money to be invested in month 1 while meeting the scheduled payments and keeping the weighted average risk at or below the specified level.

3.14 Data Envelopment Analysis

Managers often are interested in determining how efficiently various units within a company operate. Similarly, investment analysts might be interested in comparing the efficiency of several competing companies within an industry. Data Envelopment Analysis (DEA) is an LP-based methodology for performing this type of analysis. DEA determines how efficiently an operating unit (or company) converts inputs to outputs when compared with other units. We will consider how DEA may be applied via the following example.

> Mike Lister is a district manager for the Steak & Burger fast-food restaurant chain. The region Mike manages contains 12 company-owned units. Mike is in the process of evaluating the performance of these units during the past year to make recommendations on how much of an annual bonus to pay each unit's manager. He wants to base this decision, in part, on how efficiently each unit has been operated. Mike has collected the data shown in the following table on each of the 12 units. The outputs he has chosen include each unit's net profit (in $100,000s), average customer satisfaction rating, and average monthly cleanliness score. The inputs include total

labor hours (in 100,000s) and total operating costs (in \$1,000,000s). He wants to apply DEA to this data to determine an efficiency score of each unit.

	Outputs			Inputs	
Unit	**Profit**	**Satisfaction**	**Cleanliness**	**Labor Hours**	**Operating Costs**
1	5.98	7.7	92	4.74	6.75
2	7.18	9.7	99	6.38	7.42
3	4.97	9.3	98	5.04	6.35
4	5.32	7.7	87	3.61	6.34
5	3.39	7.8	94	3.45	4.43
6	4.95	7.9	88	5.25	6.31
7	2.89	8.6	90	2.36	3.23
8	6.40	9.1	100	7.09	8.69
9	6.01	7.3	89	6.49	7.28
10	6.94	8.8	89	7.36	9.07
11	5.86	8.2	93	5.46	6.69
12	8.35	9.6	97	6.58	8.75

3.14.1 DEFINING THE DECISION VARIABLES

Using DEA, the efficiency of an arbitrary unit i is defined as follows:

$$\text{Efficiency of unit } i = \frac{\text{Weighted sum of unit } i\text{'s outputs}}{\text{Weighted sum of unit } i\text{'s inputs}} = \frac{\sum_{j=1}^{n_O} \mathrm{O}_{ij}\mathrm{w}_j}{\sum_{j=1}^{n_I} \mathrm{I}_{ij}\mathrm{v}_j}$$

Here, O_{ij} represents the value of unit i on *output* j, I_{ij} represents the value of unit i on *input* j, w_j is a nonnegative weight assigned to output j, v_j is a nonnegative weight assigned to input j, n_O is the number of output variables, and n_I is the number of input variables. The problem in DEA is to determine values for the weights w_j and v_j. Thus, w_j and v_j represent the decision variables in a DEA problem.

3.14.2 DEFINING THE OBJECTIVE

A separate LP problem is solved for each unit in a DEA problem. However, for each unit the objective is the same: to maximize the weighted sum of that unit's outputs. For an arbitrary unit i, the objective is stated as:

$$\text{MAX: } \sum_{j=1}^{n_O} \mathrm{O}_{ij}\mathrm{w}_j$$

Thus, as each LP problem is solved, the unit under investigation is given the opportunity to select the best possible weights for itself (or the weights that maximize the weighted sum of its output), subject to the following constraints.

3.14.3 DEFINING THE CONSTRAINTS

It is impossible for any unit to be more than 100% efficient. So as each LP is solved, the unit under investigation cannot select weights for itself that would cause the efficiency for any unit (including itself) to be greater than 100%. Thus, for each individual unit, we require the weighted sum of the unit's outputs to be less than or equal to the weighted sum of its inputs (so the ratio of weighted outputs to weighted inputs does not exceed 100%).

$$\sum_{j=1}^{n_O} O_{kj}w_j \leq \sum_{j=1}^{n_I} I_{kj}v_j, \quad \text{for } k = 1 \text{ to the number of units}$$

or equivalently,

$$\sum_{j=1}^{n_O} O_{kj}w_j - \sum_{j=1}^{n_I} I_{kj}v_j \leq 0, \quad \text{for } k = 1 \text{ to the number of units}$$

To prevent unbounded solutions, we also require the sum of the weighted inputs for the unit under investigation (unit i) to equal one.

$$\sum_{j=1}^{n_I} I_{ij}v_j = 1$$

Because the sum of weighted inputs for the unit under investigation must equal one and its sum of the weighted outputs (being maximized) cannot exceed this value, the maximum efficiency score for the unit under investigation is also one (or 100%). Thus, units that are efficient will have a DEA efficiency score of 100%.

Important Point

When applying DEA, it is assumed that for output variables "more is better" (e.g., profit) and for input variables "less is better" (e.g., costs). Any output or input variables that do not naturally conform to these rules should be transformed before applying DEA. For example, the percentage of defective products produced is not a good choice for an output because fewer defects is actually a good thing. However, the percentage of nondefective products produced would be an acceptable choice for an output because "more is better" in that case.

3.14.4 IMPLEMENTING THE MODEL

To evaluate the efficiency of unit 1 in our example problem, we would solve the following LP problem,

MAX: $5.98w_1 + 7.7w_2 + 92w_3$ } weighted output for unit 1

Subject to: $5.98w_1 + 7.7w_2 + 92w_3 - 4.74v_1 - 6.75v_2 \leq 0$ } efficiency constraint for unit 1

$7.18w_1 + 9.7w_2 + 99w_3 - 6.38v_1 - 7.42v_2 \leq 0$ } efficiency constraint for unit 2

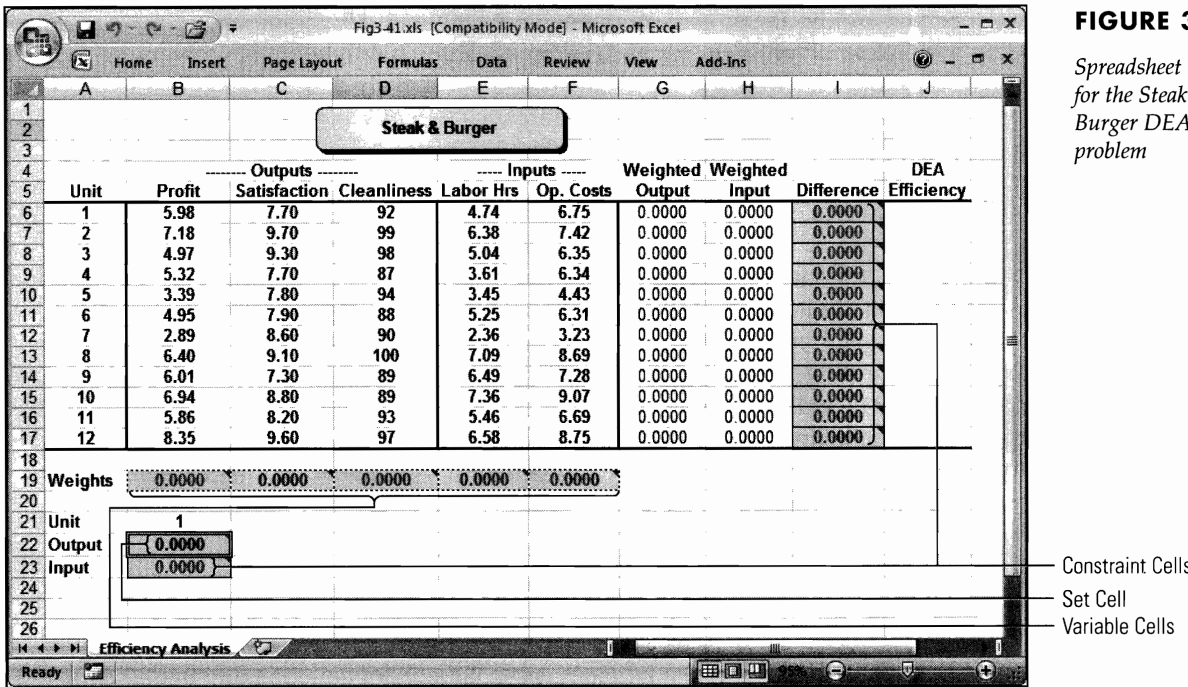
and so on to . . .

$8.35w_1 + 9.6w_2 + 97w_3 - 6.58v_1 - 8.75v_2 \leq 0$ } efficiency constraint for unit 12

$4.74v_1 + 6.75v_2 = 1$ } input constraint for unit 1

$w_1, w_2, w_3, v_1, v_2 \geq 0$ } nonnegativity conditions

A convenient way to implement this model is shown in Figure 3.41 (and in the file Fig3-41.xls on your data disk).



	-------- Outputs --------			----- Inputs -----		Weighted	Weighted		DEA
Unit	Profit	Satisfaction	Cleanliness	Labor Hrs	Op. Costs	Output	Input	Difference	Efficiency
1	5.98	7.70	92	4.74	6.75	0.0000	0.0000	0.0000	
2	7.18	9.70	99	6.38	7.42	0.0000	0.0000	0.0000	
3	4.97	9.30	98	5.04	6.35	0.0000	0.0000	0.0000	
4	5.32	7.70	87	3.61	6.34	0.0000	0.0000	0.0000	
5	3.39	7.80	94	3.45	4.43	0.0000	0.0000	0.0000	
6	4.95	7.90	88	5.25	6.31	0.0000	0.0000	0.0000	
7	2.89	8.60	90	2.36	3.23	0.0000	0.0000	0.0000	
8	6.40	9.10	100	7.09	8.69	0.0000	0.0000	0.0000	
9	6.01	7.30	89	6.49	7.28	0.0000	0.0000	0.0000	
10	6.94	8.80	89	7.36	9.07	0.0000	0.0000	0.0000	
11	5.86	8.20	93	5.46	6.69	0.0000	0.0000	0.0000	
12	8.35	9.60	97	6.58	8.75	0.0000	0.0000	0.0000	
Weights	0.0000	0.0000	0.0000	0.0000	0.0000				
Unit	1								
Output	0.0000								
Input	0.0000								

Key Cell Formulas

Cell	Formula	Copied to
G6	=SUMPRODUCT(B6:D6,$B$19:$D$19)	G7:G17
H6	=SUMPRODUCT(E6:F6,$E$19:$F$19)	H7:H17
I6	=G6-H6	I7:I17
B22	=INDEX(G6:G17,B21,1)	--
B23	=INDEX(H6:H17,B21,1)	--

FIGURE 3.41

Spreadsheet model for the Steak & Burger DEA problem

In Figure 3.41, cells B19 through F19 are reserved to represent the weights for each of the input and output variables. The weighted output for each unit is computed in column G as follows:

Formula for cell G6: =SUMPRODUCT(B6:D6,$B$19:$D$19)
(Copy to G7 through G17.)

Similarly, the weighted input for each unit is computed in column H as:

Formula for cell H6: =SUMPRODUCT(E6:F6,$E$19:$F$19)
(Copy to H7 through H17.)

The differences between the weighted outputs and weighted inputs are computed in column I. We will instruct Solver to constrain these values to be less than or equal to 0.

Formula for cell I6: =G6-H6
(Copy to I7 through I17.)

The weighted output for unit 1 (computed in cell G6) implements the appropriate objective function and could be used as the set cell for Solver in this problem. Similarly, the weighted input for unit 1 is computed in cell H6 and could be constrained to equal 1 (as specified by the input constraint for unit 1 above). However, because we need to solve a separate LP problem for each of the 12 units, it will be more convenient to handle the objective function and input constraint in a slightly different manner. To this end, we reserve cell B21 to indicate the unit number currently under investigation. Cell B22 contains a formula that returns the weighted output for this unit from the list of weighted outputs in column G.

Formula for cell B22: =INDEX(G6:G17,B21,1)

In general, the function INDEX(***range,row number,column number***) returns the value in the specified ***row number*** and ***column number*** of the given ***range***. Because cell B21 contains the number 1, the previous formula returns the value in the first row and first column of the range G6:G17—or the value in cell G6. Thus, as long as the value of cell B21 represents a valid unit number from 1 to 12, the value in cell B22 will represent the appropriate objective function for the DEA model for that unit. Similarly, the input constraint requiring the weighted inputs for the unit in question to equal 1 can be implemented in cell B23 as follows:

Formula for cell B23: =INDEX(H6:H17,B21,1)

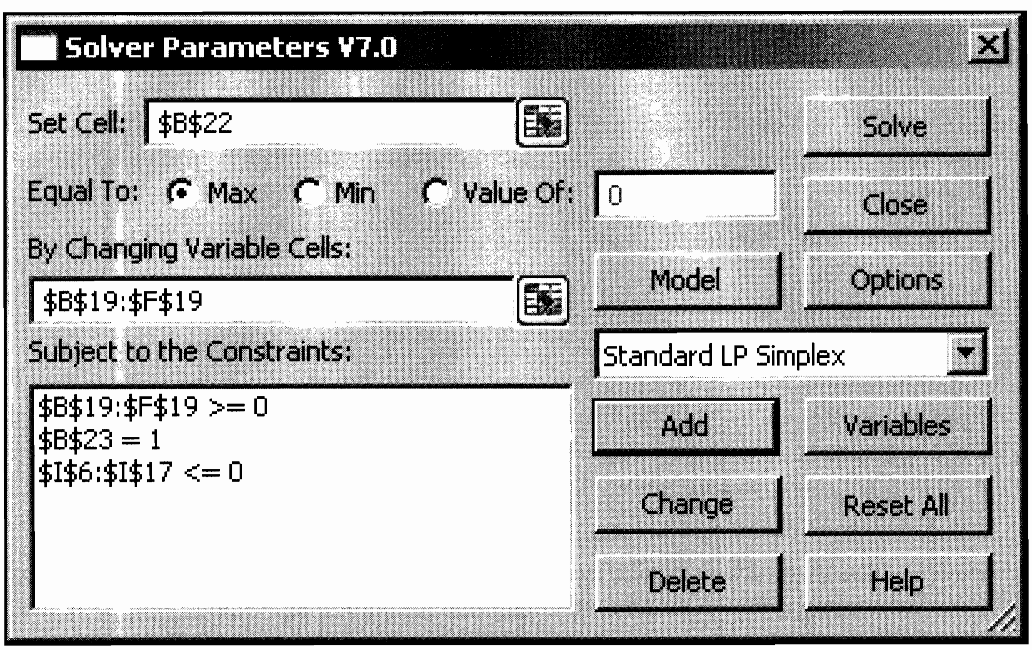
So, for whatever unit number is listed in cell B21, cell B22 represents the appropriate objective function to be maximized and cell B23 represents the weighted input that must be constrained to equal 1. This arrangement greatly simplifies the process of solving the required series of DEA models.

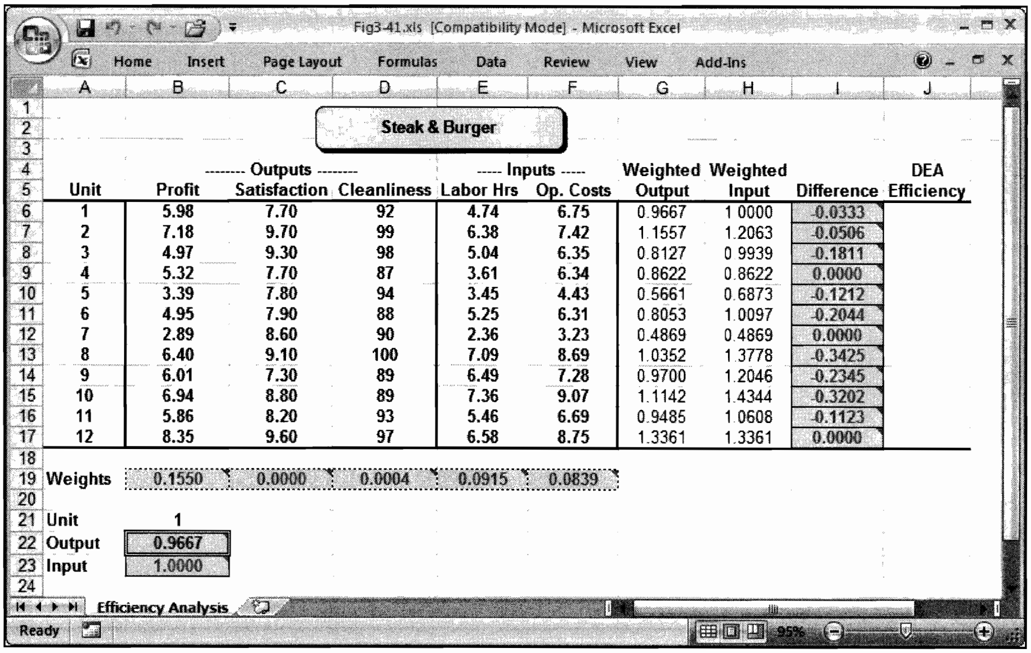
3.14.5 SOLVING THE MODEL

To solve this model, we specify the set cells, variable cells, and constraints specified in Figure 3.42. Note that exactly the same Solver settings would be used to find the optimal DEA weights for any other unit. The optimal solution for unit 1 is shown in Figure 3.43. Notice that unit 1 achieves an efficiency score of 0.9667 and therefore is slightly inefficient.

FIGURE 3.42

Solver parameters for the DEA problem





Unit	Profit (Outputs)	Satisfaction (Outputs)	Cleanliness (Outputs)	Labor Hrs (Inputs)	Op. Costs (Inputs)	Weighted Output	Weighted Input	Difference	DEA Efficiency
1	5.98	7.70	92	4.74	6.75	0.9667	1.0000	-0.0333	
2	7.18	9.70	99	6.38	7.42	1.1557	1.2063	-0.0506	
3	4.97	9.30	98	5.04	6.35	0.8127	0.9939	-0.1811	
4	5.32	7.70	87	3.61	6.34	0.8622	0.8622	0.0000	
5	3.39	7.80	94	3.45	4.43	0.5661	0.6873	-0.1212	
6	4.95	7.90	88	5.25	6.31	0.8053	1.0097	-0.2044	
7	2.89	8.60	90	2.36	3.23	0.4869	0.4869	0.0000	
8	6.40	9.10	100	7.09	8.69	1.0352	1.3778	-0.3425	
9	6.01	7.30	89	6.49	7.28	0.9700	1.2046	-0.2345	
10	6.94	8.80	89	7.36	9.07	1.1142	1.4344	-0.3202	
11	5.86	8.20	93	5.46	6.69	0.9485	1.0608	-0.1123	
12	8.35	9.60	97	6.58	8.75	1.3361	1.3361	0.0000	
Weights	0.1550	0.0000	0.0004	0.0915	0.0839				

Unit	1
Output	0.9667
Input	1.0000

FIGURE 3.43

Optimal DEA solution for unit 1

To complete the analysis for the remaining units, Mike could change the value in cell B21 manually to 2, 3, . . ., 12 and use Solver to reoptimize the worksheet for each unit and record their efficiency scores in column J. However, if there were 120 units rather than 12, this manual approach would become quite cumbersome. Fortunately, it is easy to write a simple macro in Excel to carry out this process for us automatically with the click of a button. To do this, turn on the Developer tab in the ribbon and place a command button on your worksheet (as shown in Figure 3.44) as follows:

1. Click Office button, Excel Options, Popular, Show Developer tab.
2. Click the Command Button icon on the Developer, Insert, Active X Controls menu.
3. Click and drag on your worksheet to draw a command button.

Next, we need to change a few properties of our newly created command button. To do this,

1. Click the Command Button to make sure it is selected.
2. Click the Properties icon in the Developer, Controls ribbon.

These actions cause the Properties window shown in Figure 3.45 to appear. This window lists several properties (or attributes) of the command button that you can change to customize its appearance and behavior. For present purposes, change the command button's property values as follows,

Property	New Value
(Name)	DEA
Caption	Run DEA
TakeFocusOnClick	False

FIGURE 3.44

Using the Control Toolbox to create a command button

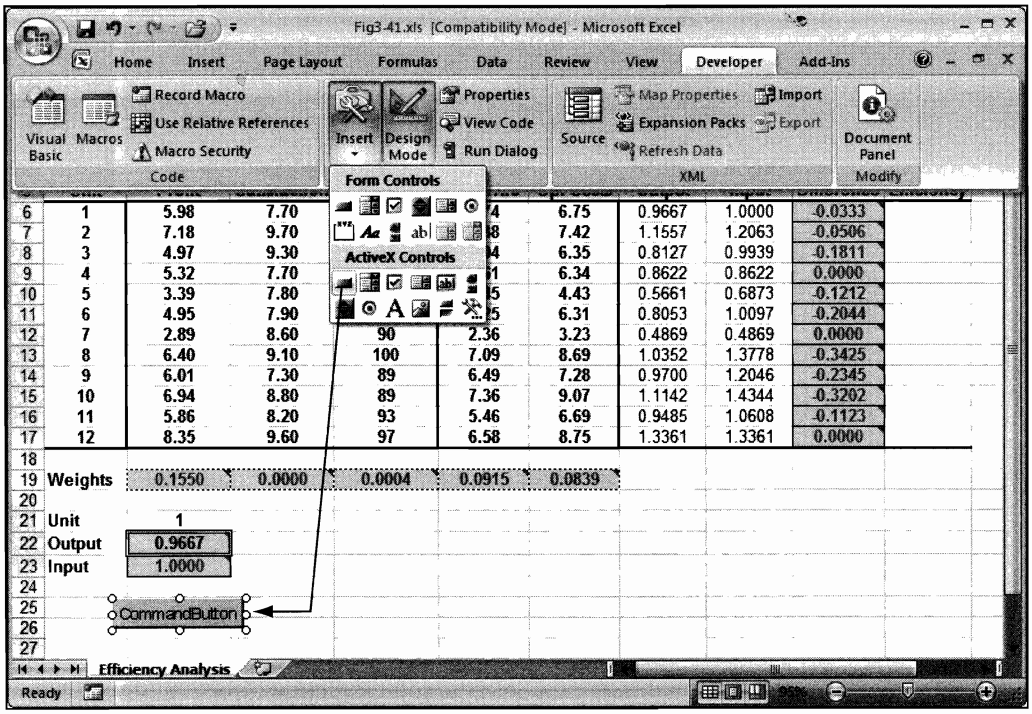
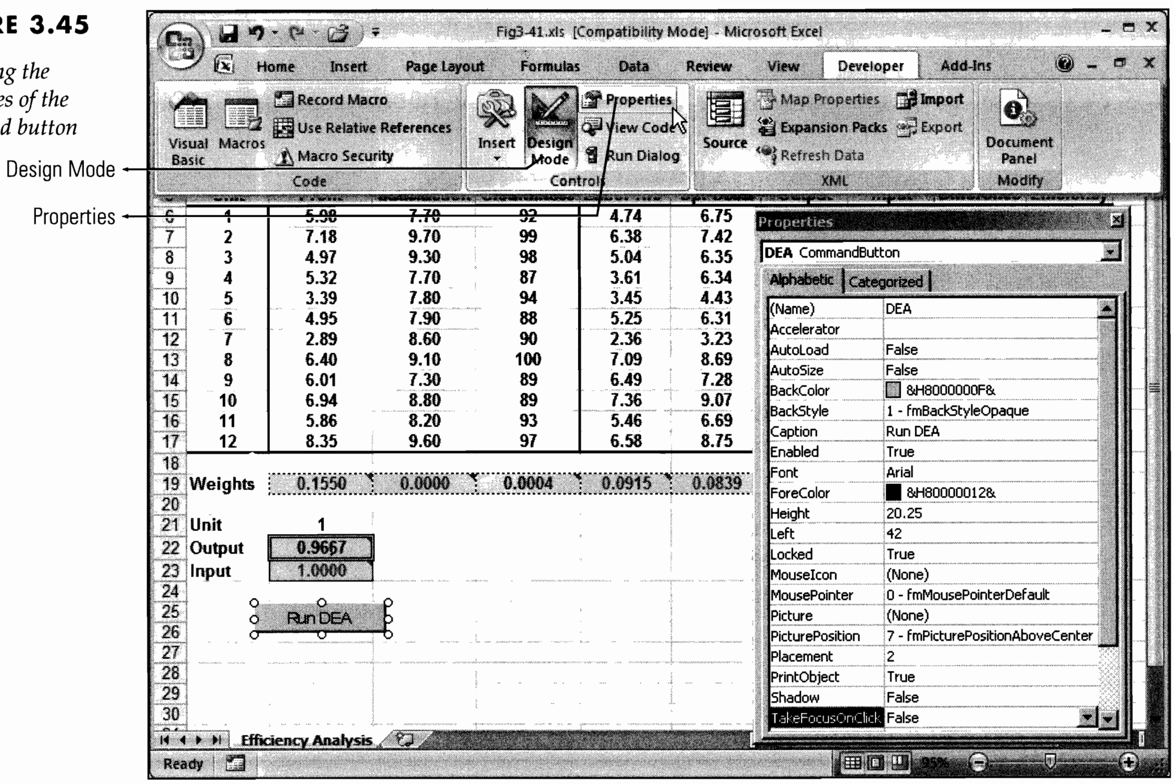


FIGURE 3.45

Adjusting the properties of the command button



Important Software Note

When you place a command button on a worksheet, it is a good idea to set its "TakeFocusOnClick" property to False. This prevents the command button from receiving the focus when it is clicked. "Focus" refers to the object on the screen that is selected or has the computer's attention. An Excel macro cannot perform certain operations if a command button has the focus.

Next, double-click the command button. This should launch Excel's Visual Basic Editor and bring up the code window for the command button's click event. Initially, the click event will not have any commands in it. Insert the statements shown in Figure 3.46. These statements will be executed whenever the command button is clicked.

Software Tip

You can toggle back and forth easily between Excel and the Visual Basic Editor by pressing Alt+F11.

If you have any programming experience, you can probably follow the logic behind the programming code listed in Figure 3.46. In a nutshell, the For and Next statements define a loop of code that will be repeated 12 times. During the first execution of the loop, the variable "unit" will equal 1. During the second execution of the loop, the variable "unit" will equal 2, and so on. During each execution of the loop, the following operations take place:

Macro Statement	Purpose
RANGE("B21")=unit	Places the current value of "unit" (the number 1, 2, 3, . . ., or 12) into cell B21 on the worksheet.
SolverSolve UserFinish:=True	Tells Solver to solve the problem without displaying the usual Solver Results dialog box.
Range("J" & 5 + unit) = Range("B22")	Takes the optimal objective function value in cell B22 and places it in row "5 + unit" (that is, row 6, 7, . . ., or 17) in column J.

To call Solver from within a macro program, we must first set a reference to the Solver.xla file. You do this from within Excel's Visual Basic editor as follows,

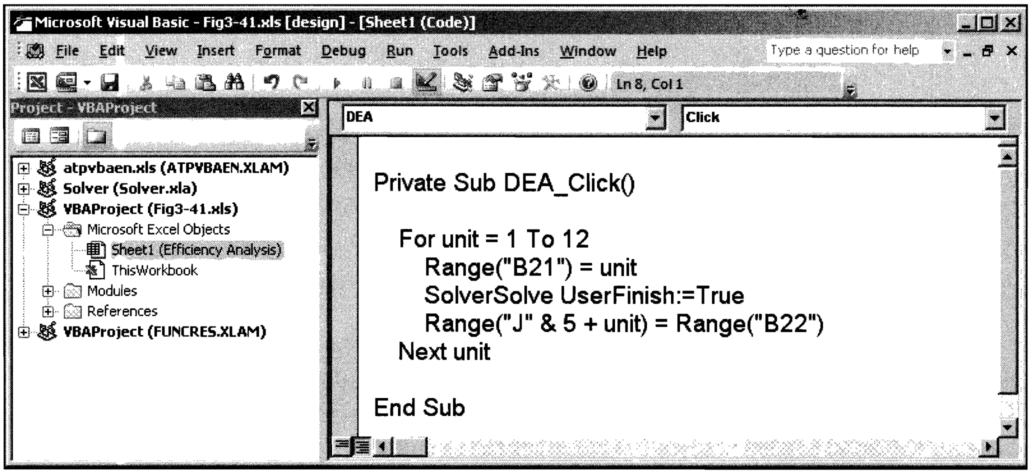
1. Click Tools, References and check the box for Solver.
2. Click OK.

We can now test our command button to see if it works. To do this,

1. Close the Visual Basic editor window (or press Alt+F11).
2. Click the Design Mode icon on the Developer, Controls ribbon.
3. Click the Run DEA command button.

FIGURE 3.46

VBA code for the command button's click event




```
Private Sub DEA_Click()

  For unit = 1 To 12
    Range("B21") = unit
    SolverSolve UserFinish:=True
    Range("J" & 5 + unit) = Range("B22")
  Next unit

End Sub
```

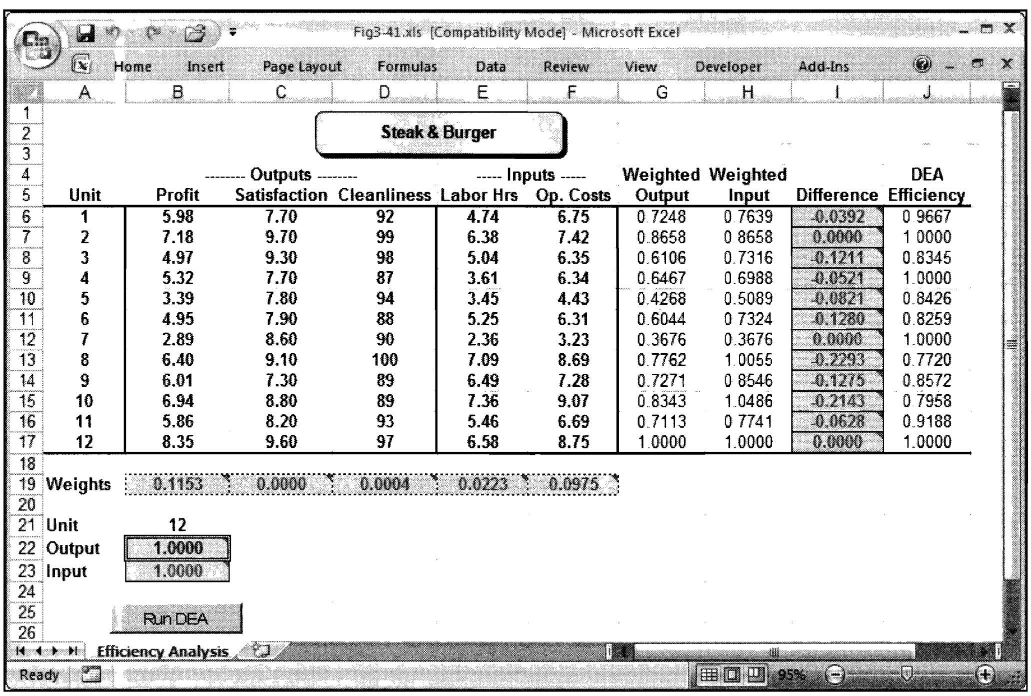
If everything worked correctly, you should see the results shown in Figure 3.47 (or see the completed file Fig3-47.xls on your data disk).

Important Software Note

It is important to remember that command buttons and other controls become operational only after you exit design mode. If the Developer, Control ribbon before clicking the Design Mode icon, you might still be in design mode and your controls might not operate correctly.

FIGURE 3.47

DEA efficiency scores for all the units



	-------- Outputs --------			----- Inputs -----		Weighted	Weighted		DEA
Unit	Profit	Satisfaction	Cleanliness	Labor Hrs	Op. Costs	Output	Input	Difference	Efficiency
1	5.98	7.70	92	4.74	6.75	0.7248	0.7639	-0.0392	0.9667
2	7.18	9.70	99	6.38	7.42	0.8658	0.8658	0.0000	1.0000
3	4.97	9.30	98	5.04	6.35	0.6106	0.7316	-0.1211	0.8345
4	5.32	7.70	87	3.61	6.34	0.6467	0.6988	-0.0521	1.0000
5	3.39	7.80	94	3.45	4.43	0.4268	0.5089	-0.0821	0.8426
6	4.95	7.90	88	5.25	6.31	0.6044	0.7324	-0.1280	0.8259
7	2.89	8.60	90	2.36	3.23	0.3676	0.3676	0.0000	1.0000
8	6.40	9.10	100	7.09	8.69	0.7762	1.0055	-0.2293	0.7720
9	6.01	7.30	89	6.49	7.28	0.7271	0.8546	-0.1275	0.8572
10	6.94	8.80	89	7.36	9.07	0.8343	1.0486	-0.2143	0.7958
11	5.86	8.20	93	5.46	6.69	0.7113	0.7741	-0.0628	0.9188
12	8.35	9.60	97	6.58	8.75	1.0000	1.0000	0.0000	1.0000
Weights	0.1153	0.0000	0.0004	0.0223	0.0975				
Unit	12								
Output	1.0000								
Input	1.0000								

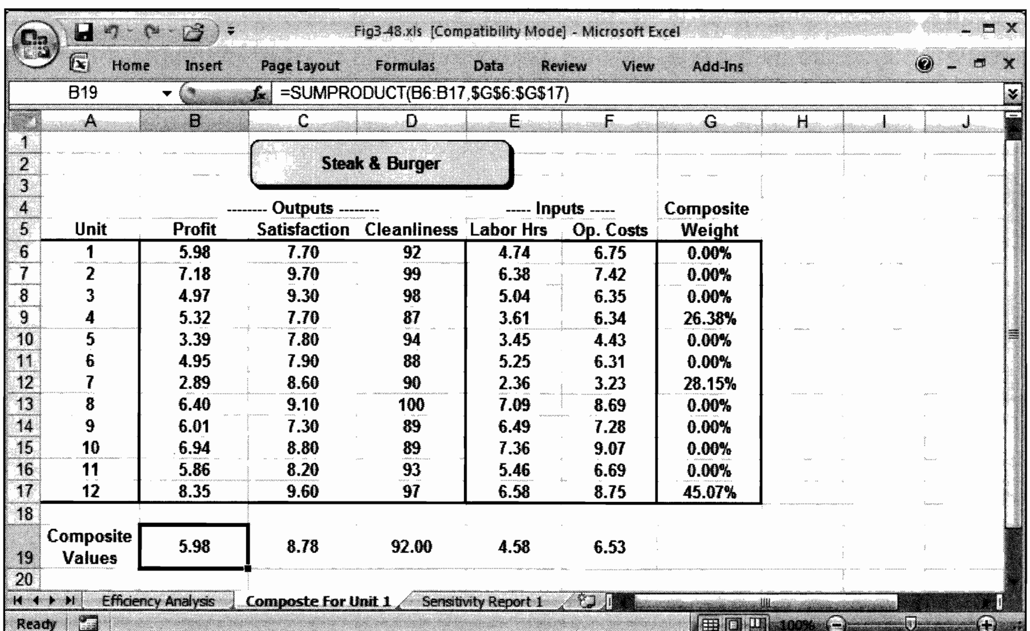
3.14.6 ANALYZING THE SOLUTION

The solution shown in Figure 3.47 indicates that units 2, 4, 7, and 12 are operating at 100% efficiency (in the DEA sense), while the remaining units are operating less efficiently. Note that an efficiency rating of 100% does not necessarily mean that a unit is operating in the best possible way. It simply means that no linear combination of the other units in the study results in a composite unit that produces at least as much output using the same or less input. On the other hand, for units that are DEA *inefficient*, there exists a linear combination of efficient units that results in a composite unit that produces at least as much output using the same or less input than the inefficient unit. The idea in DEA is that an inefficient unit should be able to operate as efficiently as this hypothetical composite unit formed from a linear combination of the efficient units.

For instance, unit 1 has an efficiency score of 96.67% and is, therefore, somewhat inefficient. Figure 3.48 (in file Fig3-48.xls on your data disk) shows that a weighted average of 26.38% of unit 4, plus 28.15% of unit 7, plus 45.07% of unit 12 produces a hypothetical composite unit with outputs greater than or equal to those of unit 1 and requiring less input than unit 1. The assumption in DEA is that unit 1 should have been able to achieve this same level of performance.

For any inefficient unit, you can determine the linear combination of efficient units that results in a more efficient composite unit as follows:

1. Solve the DEA problem for the unit in question.
2. In the Solver Results dialog box, select the Sensitivity report option.



	-------- Outputs --------			----- Inputs -----		Composite
Unit	Profit	Satisfaction	Cleanliness	Labor Hrs	Op. Costs	Weight
1	5.98	7.70	92	4.74	6.75	0.00%
2	7.18	9.70	99	6.38	7.42	0.00%
3	4.97	9.30	98	5.04	6.35	0.00%
4	5.32	7.70	87	3.61	6.34	26.38%
5	3.39	7.80	94	3.45	4.43	0.00%
6	4.95	7.90	88	5.25	6.31	0.00%
7	2.89	8.60	90	2.36	3.23	28.15%
8	6.40	9.10	100	7.09	8.69	0.00%
9	6.01	7.30	89	6.49	7.28	0.00%
10	6.94	8.80	89	7.36	9.07	0.00%
11	5.86	8.20	93	5.46	6.69	0.00%
12	8.35	9.60	97	6.58	8.75	45.07%
Composite Values	5.98	8.78	92.00	4.58	6.53	

FIGURE 3.48

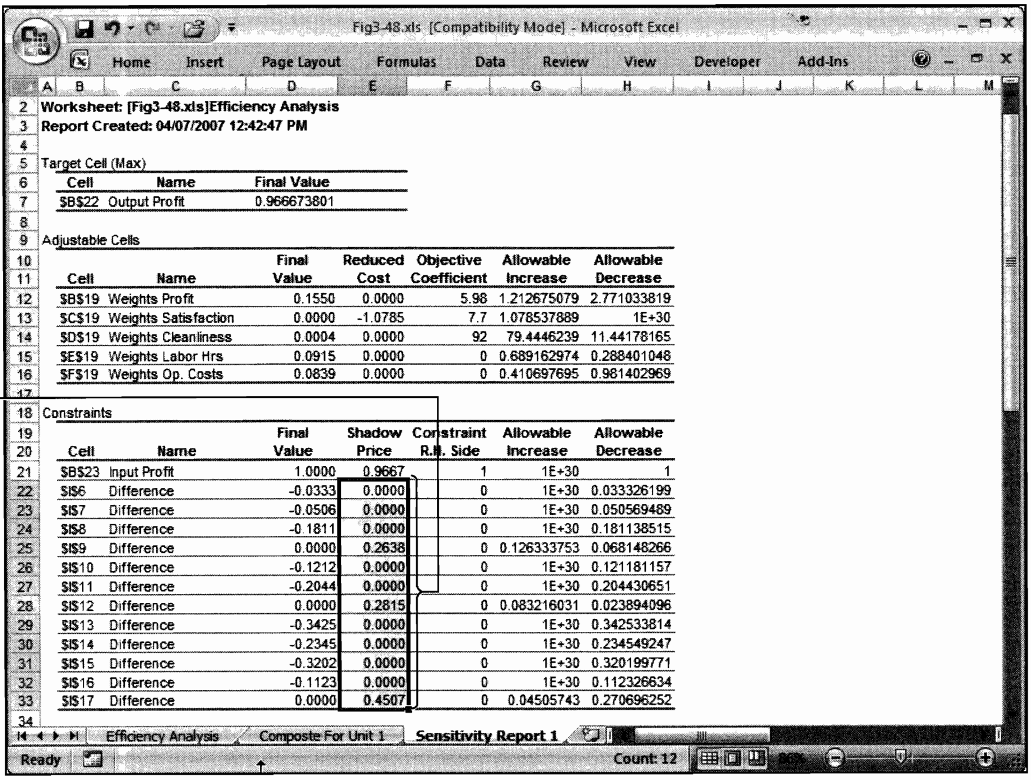
Example of a composite unit that is more efficient than unit 1

Key Cell Formulas

Cell	Formula	Copied to
B19	=SUMPRODUCT(B6:B17,$G$6:$G$17)	C19:F19

FIGURE 3.49

Sensitivity report for unit 1



Worksheet: [Fig3-48.xls]Efficiency Analysis
Report Created: 04/07/2007 12:42:47 PM

Target Cell (Max)

Cell	Name	Final Value
$B$22	Output Profit	0.966673801

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$19	Weights Profit	0.1550	0.0000	5.98	1.212675079	2.771033819
$C$19	Weights Satisfaction	0.0000	-1.0785	7.7	1.078537889	1E+30
$D$19	Weights Cleanliness	0.0004	0.0000	92	79.4446239	11.44178165
$E$19	Weights Labor Hrs	0.0915	0.0000	0	0.689162974	0.288401048
$F$19	Weights Op. Costs	0.0839	0.0000	0	0.410697695	0.981402969

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$23	Input Profit	1.0000	0.9667	1	1E+30	1
$I$6	Difference	-0.0333	0.0000	0	1E+30	0.033326199
$I$7	Difference	-0.0506	0.0000	0	1E+30	0.050569489
$I$8	Difference	-0.1811	0.0000	0	1E+30	0.181138515
$I$9	Difference	0.0000	0.2638	0	0.126333753	0.068148266
$I$10	Difference	-0.1212	0.0000	0	1E+30	0.121181157
$I$11	Difference	-0.2044	0.0000	0	1E+30	0.204430651
$I$12	Difference	0.0000	0.2815	0	0.083216031	0.023894096
$I$13	Difference	-0.3425	0.0000	0	1E+30	0.342533814
$I$14	Difference	-0.2345	0.0000	0	1E+30	0.234549247
$I$15	Difference	-0.3202	0.0000	0	1E+30	0.320199771
$I$16	Difference	-0.1123	0.0000	0	1E+30	0.112326634
$I$17	Difference	0.0000	0.4507	0	0.04505743	0.270696252

In the resulting sensitivity report, the absolute value of the Shadow Prices for the "Difference" constraints are the weights that should create a composite unit that is more efficient than the unit in question. The sensitivity report for unit 1 is shown in Figure 3.49.

Want To Know More?

To learn more about writing VBA macros in Excel, see *Excel 2007 Power Programming with VBA* by Wiley Publishing. To learn more about controlling Solver from within macros, click the Help button on the Solver Options dialog box (available in Premium Solver for Education).

3.15 Summary

This chapter described how to formulate an LP problem algebraically, implement it in a spreadsheet, and solve it using Solver. The decision variables in the algebraic formulation of a model correspond to the variable cells in the spreadsheet. The LHS formulas for each constraint in an LP model must be implemented in different cells in the spreadsheet. Also, a cell in the spreadsheet must represent the objective function in the LP model. Thus, there is a direct relationship between the various components of an algebraic formulation of an LP problem and its implementation in a spreadsheet.

There are many ways to implement a given LP problem in a spreadsheet. The process of building spreadsheet models is more an art than a science. A good spreadsheet implementation represents the problem in a way that clearly communicates its purpose, is reliable, auditable, and modifiable.

It is possible to use Excel's macro language (known as Visual Basic for Applications or VBA) to automate the process of solving LP models. This is particularly useful in problems in which an analyst might want to solve several related problems in succession.

3.16 References

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THE WORLD OF MANAGEMENT SCIENCE

Optimizing Production, Inventory, and Distribution at Kellogg

The Kellogg Company (http://www.kelloggs.com) is the largest cereal producer in the world and a leading producer of convenience foods. In 1999, Kellogg's worldwide sales totaled nearly $7 billion. Kellogg operates five plants in the United States and Canada and has seven core distribution centers and roughly fifteen co-packers that contract to produce or pack some of Kellogg's products. In the cereal business alone, Kellogg must coordinate the production of 80 products while inventorying and distributing over 600 stock keeping units with roughly 90 production lines and 180 packaging lines. Optimizing this many decision variables is obviously a daunting challenge.

Since 1990, Kellogg has been using a large-scale, multiperiod linear program, called the Kellogg Planning System (KPS), to guide production and distribution decisions. Most large companies like Kellogg employ some sort of enterprise resource planning (ERP). Kellogg's ERP systems is a largely custom, home-grown product, and KPS is a custom-developed tool to complement the ERP system.

An operational-level version of KPS is used at a weekly level of detail to help determine where products are produced and how finished products and in-process products are shipped between plants and distribution centers. A tactical-level version of KPS is used at a monthly level of detail to help establish plant budgets and make capacity and consolidation decisions. Kellogg attributes annual savings of $40–$45 million to the use of the KPS system.

Source: Brown, G., J. Keegan, B. Vigus, and K. Wood, "The Kellogg Company Optimizes Production, Inventory, and Distribution," *Interfaces*, Vol. 35, No. 6, 2001.

Questions and Problems

1. In creating the spreadsheet models for the problems in this chapter, cells in the spreadsheets had to be reserved to represent each of the decision variables in the algebraic models. We reserved these cells in the spreadsheets by entering values of zero in them. Why didn't we place some other value or formula in these cells? Would doing so have made any difference?
2. Four goals should be considered when trying to design an effective spreadsheet model: communication, reliability, auditability, and maintainability. We also noted that a spreadsheet design that results in formulas that can be copied is usually more effective than other designs. Briefly describe how using formulas that can be copied supports the four spreadsheet modeling goals.
3. Refer to question 13 at the end of Chapter 2. Implement a spreadsheet model for this problem and solve it using Solver.
4. Refer to question 14 at the end of Chapter 2. Implement a spreadsheet model for this problem and solve it using Solver.
5. Refer to question 17 at the end of Chapter 2. Implement a spreadsheet model for this problem and solve it using Solver.
6. Refer to question 18 at the end of Chapter 2. Implement a spreadsheet model for this problem and solve it using Solver.
7. Refer to question 19 at the end of Chapter 2. Implement a spreadsheet model for this problem and solve it using Solver.
8. Refer to question 20 at the end of Chapter 2. Implement a spreadsheet model for this problem and solve it using Solver.
9. Refer to question 21 at the end of Chapter 2. Implement a spreadsheet model for this problem and solve it using Solver.
10. Refer to question 22 at the end of Chapter 2. Implement a spreadsheet model for this problem and solve it using Solver.
11. Refer to question 23 at the end of Chapter 2. Implement a spreadsheet model for this problem and solve it using Solver.
12. A furniture manufacturer produces two types of tables (country and contemporary) using three types of machines. The time required to produce the tables on each machine is given in the following table.

Machine	**Country**	**Contemporary**	**Total Machine Time Available per Week**
Router	1.5	2.0	1,000
Sander	3.0	4.5	2,000
Polisher	2.5	1.5	1,500

Country tables sell for \$350 and contemporary tables sell for \$450. Management has determined that at least 20% of the tables made should be country and at least 30% should be contemporary. How many of each type of table should the company produce if it wants to maximize its revenue?

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?
d. How will your spreadsheet model differ if there are 25 types of tables and 15 machine processes involved in manufacturing them?

13. Aire-Co produces home dehumidifiers at two different plants in Atlanta and Phoenix. The per unit cost of production in Atlanta and Phoenix is \$400 and \$360, respectively. Each plant can produce a maximum of 300 units per month. Inventory

holding costs are assessed at $30 per unit in beginning inventory each month. Aire-Co estimates the demand for its product to be 300, 400, and 500 units, respectively, over the next three months. Aire-Co wants to be able to meet this demand at minimum cost.

a. Formulate an LP model for this problem.
b. Implement your model in a spreadsheet and solve it.
c. What is the optimal solution?
d. How does the solution change if each plant is required to produce at least 50 units per month?
e. How does the solution change if each plant is required to produce at least 100 units per month?

14. The Molokai Nut Company (MNC) makes four different products from macadamia nuts grown in the Hawaiian Islands: chocolate-coated whole nuts (Whole), chocolate-coated nut clusters (Cluster), chocolate-coated nut crunch bars (Crunch), and plain roasted nuts (Roasted). The company is barely able to keep up with the increasing demand for these products. However, increasing raw material prices and foreign competition are forcing MNC to watch its margins to ensure that it is operating in the most efficient manner possible. To meet marketing demands for the coming week, MNC needs to produce at least 1,000 pounds of the Whole product, between 400 and 500 pounds of the Cluster product, no more than 150 pounds of the Crunch product, and no more than 200 pounds of Roasted product. Each pound of the Whole, Cluster, Crunch, and Roasted product contains, respectively, 60%, 40%, 20%, and 100% macadamia nuts, with the remaining weight made up of chocolate coating. The company has 1,100 pounds of nuts and 800 pounds of chocolate available for use in the next week. The various products are made using four different machines that hull the nuts, roast the nuts, coat the nuts in chocolate (if needed), and package the products. The following table summaries the time required by each product on each machine. Each machine has 60 hours of time available in the coming week.

	Minutes Required per Pound			
Machine	**Whole**	**Cluster**	**Crunch**	**Roasted**
Hulling	1.00	1.00	1.00	1.00
Roasting	2.00	1.50	1.00	1.75
Coating	1.00	0.70	0.20	0.00
Packaging	2.50	1.60	1.25	1.00

The selling price and variable cost associated with each pound of product is summarized below:

	Per Pound Revenue and Costs			
	Whole	**Cluster**	**Crunch**	**Roasted**
Selling Price	$5.00	$4.00	$3.20	$4.50
Variable Cost	$3.15	$2.60	$2.16	$3.10

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

15. A company is trying to determine how to allocate its $145,000 advertising budget for a new product. The company is considering newspaper ads and television commercials as its primary means for advertising. The following table summarizes the costs

of advertising in these different media and the number of new customers reached by increasing amounts of advertising.

Media & # of Ads	# of New Customers Reached	Cost per Ad
Newspaper: 1–10	900	$1,000
Newspaper: 11–20	700	$900
Newspaper: 21–30	400	$800
Television: 1–5	10,000	$12,000
Television: 6–10	7,500	$10,000
Television: 11–15	5,000	$8,000

For instance, each of the first ten ads the company places in newspapers will cost $1,000 and is expected to reach 900 new customers. Each of the next 10 newspaper ads will cost $900 and is expected to reach 700 new customers. Note that the number of new customers reached by increasing amounts of advertising decreases as the advertising saturates the market. Assume that the company will purchase no more than 30 newspaper ads and no more than 15 television ads.

a. Formulate an LP model for this problem to maximize the number of new customers reached by advertising.
b. Implement your model in a spreadsheet and solve it.
c. What is the optimal solution?
d. Suppose the number of new customers reached by 11–20 newspaper ads is 400 and the number of new customers reached by 21–30 newspaper ads is 700. Make these changes in your spreadsheet and reoptimize the problem. What is the new optimal solution? What (if anything) is wrong with this solution and why?

16. The Shop at Home Network sells various household goods during live television broadcasts. The company owns several warehouses to hold many of the goods it sells, but also leases extra warehouse space when needed. During the next five months the company expects that it will need to lease the following amounts of extra warehouse space:

Month	1	2	3	4	5
Square Feet Needed	20,000	30,000	40,000	35,000	50,000

At the beginning of any month the company can lease extra space for one or more months at the following costs:

Lease Term (months)	1	2	3	4	5
Cost per Sq. Ft. Leased	$55	$95	$130	$155	$185

So, for instance, at the start of month 1 the company can lease as much space as it wants for 4 months at a cost of $155 per square foot. Similarly, at the start of month 3 it can lease any amount of space for 2 months at a cost of $95 per square foot. The company wants to determine the least costly way of meeting its warehousing needs over the coming 5 months.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?
d. How much would it cost the company to meet its space needs if in each month it leases for one month exactly the amount of space required for the month?

17. A bank has $650,000 in assets to allocate among investments in bonds, home mortgages, car loans, and personal loans. Bonds are expected to produce a return of 10%, mortgages 8.5%, car loans 9.5%, and personal loans 12.5%. To make sure the portfolio is not too risky, the bank wants to restrict personal loans to no more than the 25% of the total portfolio. The bank also wants to ensure that more money is invested in mortgages than in personal loans. It also wants to invest more in bonds than personal loans.
    a. Formulate an LP model for this problem with the objective of maximizing the expected return on the portfolio.
    b. Implement your model in a spreadsheet and solve it.
    c. What it the optimal solution?
18. Valu-Com Electronics manufactures five different models of telecommunications interface cards for personal and laptop computers. As summarized in the following table, each of these devices requires differing amounts of printed circuit (PC) board, resistors, memory chips, and assembly.

	Per Unit Requirements				
	HyperLink	**FastLink**	**SpeedLink**	**MicroLink**	**EtherLink**
PC Board (square inches)	20	15	10	8	5
Resistors	28	24	18	12	16
Memory Chips	8	8	4	4	6
Assembly Labor (in hours)	0.75	0.6	0.5	0.65	1

The unit wholesale price and manufacturing cost for each model are as follows.

	Per Unit Revenues and Costs				
	HyperLink	**FastLink**	**SpeedLink**	**MicroLink**	**EtherLink**
Wholesale Price	$189	$149	$129	$169	$139
Manufacturing Cost	$136	$101	$ 96	$137	$101

In their next production period, Valu-Com has 80,000 square inches of PC board, 100,000 resistors, 30,000 memory chips, and 5,000 hours of assembly time available. The company can sell all the product it can manufacture, but the marketing department wants to be sure that it produces at least 500 units of each product and at least twice as many FastLink cards as HyperLink cards while maximizing profit.
    a. Formulate an LP model for this problem.
    b. Create a spreadsheet model for this problem and solve it using Solver.
    c. What is the optimal solution?
    d. Could Valu-Com make more money if it schedules its assembly workers to work overtime?
19. A trust officer at the Blacksburg National Bank needs to determine how to invest $100,000 in the following collection of bonds to maximize the annual return.

Bond	Annual Return	Maturity	Risk	Tax-Free
A	9.5%	Long	High	Yes
B	8.0%	Short	Low	Yes
C	9.0%	Long	Low	No
D	9.0%	Long	High	Yes
E	9.0%	Short	High	No

The officer wants to invest at least 50% of the money in short-term issues and no more than 50% in high-risk issues. At least 30% of the funds should go into tax-free investments and at least 40% of the total annual return should be tax-free.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

20. The Weedwacker Company manufactures two types of lawn trimmers: an electric model and a gas model. The company has contracted to supply a national discount retail chain with a total of 30,000 electric trimmers and 15,000 gas trimmers. However, Weedwacker's production capability is limited in three departments: production, assembly, and packaging. The following table summarizes the hours of processing time available and the processing time required by each department, for both types of trimmers:

	Hours Required per Trimmer		
	Electric	**Gas**	**Hours Available**
Production	0.20	0.40	10,000
Assembly	0.30	0.50	15,000
Packaging	0.10	0.10	5,000

The company makes its electric trimmer in-house for \$55 and its gas trimmer for \$85. Alternatively, it can buy electric and gas trimmers from another source for \$67 and \$95, respectively. How many gas and electric trimmers should Weedwacker make and how many should it buy from its competitor to fulfill its contract in the least costly manner?

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

21. A manufacturer of prefabricated homes has decided to subcontract four components of the homes. Several companies are interested in receiving this business, but none can handle more than one subcontract. The bids made by the companies for the various subcontracts are summarized in the following table.

Bids by Companies (in \$1,000s) for Various Subcontracts

	Company			
Component	**A**	**B**	**C**	**D**
1	185	225	193	207
2	200	190	175	225
3	330	320	315	300
4	375	389	425	445

Assuming that all the companies can perform each subcontract equally well, to which company should each subcontract be assigned if the home manufacturer wants to minimize payments to the subcontractors?

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

22. Tarmac Chemical Corporation produces a special chemical compound—called CHEMIX—that is used extensively in high school chemistry classes. This compound must contain at least 20% sulfur, at least 30% iron oxide, and at least 30% but no

more than 45% potassium. Tarmac's marketing department has estimated that it will need at least 600 pounds of this compound to meet the expected demand during the coming school session. Tarmac can buy three compounds to mix together to produce CHEMIX. The makeup of these compounds is shown in the following table.

Compound	Sulfur	Iron Oxide	Potassium
1	20%	60%	20%
2	40%	30%	30%
3	10%	40%	50%

Compounds 1, 2, and 3 cost $5.00, $5.25, and $5.50 per pound, respectively. Tarmac wants to use an LP model to determine the least costly way of producing enough CHEMIX to meet the demand expected for the coming year.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

23. Holiday Fruit Company buys oranges and processes them into gift fruit baskets and fresh juice. The company grades the fruit it buys on a scale from 1 (lowest quality) to 5 (highest quality). The following table summarizes Holiday's current inventory of fruit.

Grade	Supply (1000s of lbs)
1	90
2	225
3	300
4	100
5	75

Each pound of oranges devoted to fruit baskets results in a marginal profit of $2.50, whereas each pound devoted to fresh juice results in a marginal profit of $1.75. Holiday wants the fruit in its baskets to have an average quality grade of at least 3.75 and its fresh juice to have a average quality grade of at least 2.50.

a. Formulate an optimization model for this problem.
b. Implement your model in a spreadsheet and solve it.
c. What it the optimal solution?

24. Riverside Oil Company in eastern Kentucky produces regular and supreme gasoline. Each barrel of regular sells for $21 and must have an octane rating of at least 90. Each barrel of supreme sells for $25 and must have an octane rating of at least 97. Each of these types of gasoline are manufactured by mixing different quantities of the following three inputs:

Input	Cost per Barrel	Octane Rating	Barrels Available (in 1000s)
1	$17.25	100	150
2	$15.75	87	350
3	$17.75	110	300

Riverside has orders for 300,000 barrels of regular and 450,000 barrels of supreme. How should the company allocate the available inputs to the production of regular and supreme gasoline if it wants to maximize profits?

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

25. Maintenance at a major theme park in central Florida is an ongoing process that occurs 24 hours a day. Because it is a long drive from most residential areas to the park, employees do not like to work shifts of fewer than eight hours. These 8-hour shifts start every four hours throughout the day. The number of maintenance workers needed at different times throughout the day varies. The following table summarizes the minimum number of employees needed in each 4-hour time period.

Time Period	Minimum Employees Needed
12 a.m. to 4 a.m.	90
4 a.m. to 8 a.m.	215
8 a.m. to 12 p.m.	250
12 p.m. to 4 p.m.	165
4 p.m. to 8 p.m.	300
8 p.m. to 12 a.m.	125

The maintenance supervisor wants to determine the minimum number of employees to schedule that meets the minimum staffing requirements.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

26. Radmore Memorial Hospital has a problem in its fluids analysis lab. The lab has available three machines that analyze various fluid samples. Recently, the demand for analyzing blood samples has increased so much that the lab director is having difficulty getting all the samples analyzed quickly enough and still completing the other fluid work that comes into the lab. The lab works with five types of blood specimens. Any machine can be used to process any of the specimens. However, the amount of time required by each machine varies depending on the type of specimen being analyzed. These times are summarized in the following table.

Required Specimen Processing Time in Minutes

	Specimen Type				
Machine	1	2	3	4	5
A	3	4	4	5	3
B	5	3	5	4	5
C	2	5	3	3	4

Each machine can be used a total of 8 hours a day. Blood samples collected on a given day arrive at the lab and are stored overnight and processed the next day. So, at the beginning of each day, the lab director must determine how to allocate the various samples to the machines for analysis. This morning, the lab has 80 type-1 specimens, 75 type-2 specimens, 80 type-3 specimens, 120 type-4 specimens, and 60 type-5 specimens awaiting processing. The lab director wants to know how many of each type of specimen should be analyzed on each machine to minimize the total time that the machines are devoted to analyzing blood samples.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?
d. How much processing time will be available on each machine if this solution is implemented?
e. How would the model and solution change if the lab director wanted to balance the use of each machine so that each machine were used approximately the same amount of time?

27. Virginia Tech operates its own power generating plant. The electricity generated by this plant supplies power to the university and to local businesses and residences in the Blacksburg area. The plant burns three types of coal, which produce steam that drives the turbines that generate the electricity. The Environmental Protection Agency (EPA) requires that for each ton of coal burned, the emissions from the coal furnace smoke stacks contain no more than 2,500 parts per million (ppm) of sulfur and no more than 2.8 kilograms (kg) of coal dust. The following table summarizes the amounts of sulfur, coal dust, and steam that result from burning a ton of each type of coal.

Coal	Sulfur (in ppm)	Coal Dust (in kg)	Pounds of Steam Produced
1	1,100	1.7	24,000
2	3,500	3.2	36,000
3	1,300	2.4	28,000

The three types of coal can be mixed and burned in any combination. The resulting emission of sulfur or coal dust and the pounds of steam produced by any mixture are given as the weighted average of the values shown in the table for each type of coal. For example, if the coals are mixed to produce a blend that consists of 35% of coal 1, 40% of coal 2, and 25% of coal 3, the sulfur emission (in ppm) resulting from burning one ton of this blend is:

$$0.35 \times 1{,}100 + 0.40 \times 3{,}500 + 0.25 \times 1{,}300 = 2{,}110$$

The manager of this facility wants to determine the blend of coal that will produce the maximum pounds of steam per ton without violating the EPA requirements.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?
d. If the furnace can burn up to 30 tons of coal per hour, what is the maximum amount of steam that can be produced per hour?

28. Kentwood Electronics manufactures three components for stereo systems: CD players, tape decks, and stereo tuners. The wholesale price and manufacturing cost of each item are shown in the following table.

	Wholesale	Manufacturing
Component	**Price**	**Cost**
CD Player	$150	$75
Tape Deck	$85	$35
Stereo Tuner	$70	$30

Each CD player produced requires 3 hours of assembly; each tape deck requires 2 hours of assembly; and each tuner requires 1 hour of assembly. The marketing department has indicated that it can sell no more than 150,000 CD players, 100,000 tape decks, and 90,000 stereo tuners. However, the demand is expected to be at least 50,000 units of each item, and Kentwood wants to meet this demand. If Kentwood has 400,000 hours of assembly time available, how many CD players, tape decks, and stereo tuners should it produce to maximize profits while meeting the minimum demand figures?

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

29. The Rent-A-Dent car rental company allows its customers to pick up a rental car at one location and return it to any of its locations. Currently, two locations (1 and 2) have 16 and 18 surplus cars, respectively, and four locations (3, 4, 5, and 6) each need 10 cars. The costs of getting the surplus cars from locations 1 and 2 to the other locations are summarized in the following table.

Costs of Transporting Cars Between Locations

	Location 3	Location 4	Location 5	Location 6
Location 1	\$54	\$17	\$23	\$30
Location 2	\$24	\$18	\$19	\$31

Because 34 surplus cars are available at locations 1 and 2, and 40 cars are needed at locations 3, 4, 5, and 6, some locations will not receive as many cars as they need. However, management wants to make sure that all the surplus cars are sent where they are needed, and that each location needing cars receives at least five.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

30. The Sentry Lock Corporation manufactures a popular commercial security lock at plants in Macon, Louisville, Detroit, and Phoenix. The per unit cost of production at each plant is \$35.50, \$37.50, \$39.00, and \$36.25, respectively, and the annual production capacity at each plant is 18,000, 15,000, 25,000, and 20,000, respectively. Sentry's locks are sold to retailers through wholesale distributors in seven cities across the United States. The unit cost of shipping from each plant to each distributor is summarized in the following table along with the forecasted demand from each distributor for the coming year.

Unit Shipping Cost to Distributor in

Plants	Tacoma	San Diego	Dallas	Denver	St. Louis	Tampa	Baltimore
Macon	\$2.50	\$2.75	\$1.75	\$2.00	\$2.10	\$1.80	\$1.65
Louisville	\$1.85	\$1.90	\$1.50	\$1.60	\$1.00	\$1.90	\$1.85
Detroit	\$2.30	\$2.25	\$1.85	\$1.25	\$1.50	\$2.25	\$2.00
Phoenix	\$1.90	\$0.90	\$1.60	\$1.75	\$2.00	\$2.50	\$2.65
Demand	8,500	14,500	13,500	12,600	18,000	15,000	9,000

Sentry wants to determine the least expensive way of manufacturing and shipping locks from their plants to the distributors. Because the total demand from distributors exceeds the total production capacity for all the plants, Sentry realizes it will not be able to satisfy all the demand for its product, but wants to make sure each distributor will have the opportunity to fill at least 80% of the orders they receive.

a. Create a spreadsheet model for this problem and solve it.
b. What is the optimal solution?

31. A paper recycling company converts newspaper, mixed paper, white office paper, and cardboard into pulp for newsprint, packaging paper, and print stock quality paper. The following table summarizes the yield for each kind of pulp recovered from each ton of recycled material.

Recycling Yield

	Newsprint	Packaging	Print Stock
Newspaper	85%	80%	—
Mixed Paper	90%	80%	70%
White Office Paper	90%	85%	80%
Cardboard	80%	70%	—

For instance, a ton of newspaper can be recycled using a technique that yields 0.85 tons of newsprint pulp. Alternatively, a ton of newspaper can be recycled using a technique that yields 0.80 tons of packaging paper. Similarly, a ton of cardboard can be recycled to yield 0.80 tons of newsprint or 0.70 tons of packaging paper pulp. Note that newspaper and cardboard cannot be converted to print stock pulp using the techniques available to the recycler.

The cost of processing each ton of raw material into the various types of pulp is summarized in the following table, along with the amount of each of the four raw materials that can be purchased and their costs.

	Processing Costs per Ton				
	Newsprint	**Packaging**	**Print Stock**	**Purchase Cost Per Ton**	**Tons Available**
Newspaper	$6.50	$11.00	—	$15	600
Mixed Paper	$9.75	$12.25	$9.50	$16	500
White Office Paper	$4.75	$7.75	$8.50	$19	300
Cardboard	$7.50	$8.50	—	$17	400

The recycler wants to determine the least costly way of producing 500 tons of newsprint pulp, 600 tons of packaging paper pulp, and 300 tons of print stock quality pulp.

a. Create a spreadsheet model for this problem and solve it.
b. What is the optimal solution?

32. A winery has the following capacity to produce an exclusive dinner wine at either of its two vineyards at the indicated costs:

Vineyard	Capacity	Cost per Bottle
1	3,500 bottles	$23
2	3,100 bottles	$25

Four Italian restaurants around the country are interested in purchasing this wine. Because the wine is exclusive, they all want to buy as much as they need but will take whatever they can get. The maximum amounts required by the restaurants and the prices they are willing to pay are summarized in the following table.

Restaurant	Maximum Demand	Price
1	1,800 bottles	$69
2	2,300 bottles	$67
3	1,250 bottles	$70
4	1,750 bottles	$66

The costs of shipping a bottle from the vineyards to the restaurants are summarized in the following table.

	Restaurant			
Vineyard	**1**	**2**	**3**	**4**
1	$7	$8	$13	$9
2	$12	$6	$8	$7

The winery needs to determine the production and shipping plan that allows it to maximize its profits on this wine.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

33. The Pitts Barbecue Company makes three kinds of barbecue sauce: Extra Hot, Hot, and Mild. Pitts' vice president of marketing estimates that the company can sell 8,000 cases of its Extra Hot sauce plus 10 extra cases for every dollar it spends promoting this sauce; 10,000 cases of Hot sauce plus 8 extra cases for every dollar spent promoting this sauce; and 12,000 cases of its Mild sauce plus 5 extra cases for every dollar spent promoting this sauce. Although each barbecue sauce sells for $10 per case, the cost of producing the different types of sauce varies. It costs the company $6 to produce a case of Extra Hot sauce, $5.50 to produce a case of Hot sauce, and $5.25 to produce a case of Mild sauce. The president of the company wants to make sure the company manufactures at least the minimum amounts of each sauce that the marketing vice president thinks the company can sell. A budget of $25,000 total has been approved for promoting these items, of which at least $5,000 must be spent advertising each item. How many cases of each type of sauce should be made and how do you suggest that the company allocate the promotional budget if it wants to maximize profits?
    a. Formulate an LP model for this problem.
    b. Create a spreadsheet model for this problem and solve it using Solver.
    c. What is the optimal solution?
34. Acme Manufacturing makes a variety of household appliances at a single manufacturing facility. The expected demand for one of these appliances during the next four months is shown in the following table along with the expected production costs and the expected capacity for producing these items.

	Month			
	1	**2**	**3**	**4**
Demand	420	580	310	540
Production Cost	$49.00	$45.00	$46.00	$47.00
Production Capacity	500	520	450	550

Acme estimates that it costs $1.50 per month for each unit of this appliance carried in inventory (estimated by averaging the beginning and ending inventory levels each month). Currently, Acme has 120 units in inventory on hand for this product. To maintain a level workforce, the company wants to produce at least 400 units per month. It also wants to maintain a safety stock of at least 50 units per month. Acme wants to determine how many of each appliance to manufacture during each of the next four months to meet the expected demand at the lowest possible total cost.
    a. Formulate an LP model for this problem.
    b. Create a spreadsheet model for this problem and solve it using Solver.
    c. What is the optimal solution?
    d. How much money could Acme save if it were willing to drop the restriction about producing at least 400 units per month?
35. Paul Bergey is in charge of loading cargo ships for International Cargo Company (ICC) at the port in Newport News, Virginia. Paul is preparing a loading plan for an ICC freighter destined for Ghana. An agricultural commodities dealer wants to transport the following products aboard this ship.

Commodity	Amount Available (tons)	Volume per Ton (cubic feet)	Profit per Ton ($)
1	4,800	40	70
2	2,500	25	50
3	1,200	60	60
4	1,700	55	80

Paul can elect to load any and/or all of the available commodities. However, the ship has three cargo holds with the following capacity restrictions:

Cargo Hold	Weight Capacity (tons)	Volume Capacity (cubic feet)
Forward	3,000	145,000
Center	6,000	180,000
Rear	4,000	155,000

More than one type of commodity can be placed in the same cargo hold. However, because of balance considerations, the weight in the forward cargo hold must be within 10% of the weight in the rear cargo hold and the center cargo hold must be between 40% to 60% of the total weight on board.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

36. The Pelletier Corporation has just discovered that it will not have enough warehouse space for the next five months. The additional warehouse space requirements for this period are:

Month	1	2	3	4	5
Additional Space Needed (in 1000 sq ft)	25	10	20	10	5

To cover its space requirements, the firm plans to lease additional warehouse space on a short-term basis. Over the next five months, a local warehouse has agreed to lease Pelletier any amount of space for any number of months according to the following cost schedule.

Length of Lease (in months)	1	2	3	4	5
Cost per 1000 square feet	\$300	\$525	\$775	\$850	\$975

This schedule of leasing options is available to Pelletier at the beginning of each of the next five months. For example, the company could elect to lease 5,000 square feet for 4 months beginning in month 1 (at a cost of \$850 $\times$ 5) and lease 10,000 square feet for 2 months beginning in month 3 (at a cost of \$525 $\times$ 10).

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

37. Carter Enterprises is involved in the soybean business in South Carolina, Alabama, and Georgia. The president of the company, Earl Carter, goes to a commodity sale once a month where he buys and sells soybeans in bulk. Carter uses a local warehouse for storing his soybean inventory. This warehouse charges \$10 per average ton of soybeans stored per month (based on the average of the beginning and ending inventory each month). The warehouse guarantees Carter the capacity to store up to 400 tons of soybeans at the end of each month. Carter has estimated what he believes the price per ton of soybeans will be during each of the next six months. These prices are summarized in the following table.

Month	1	2	3	4	5	6
Price per Ton	\$135	\$110	\$150	\$175	\$130	\$145

Assume that Carter currently has 70 tons of soybeans stored in the warehouse. How many tons of soybeans should Carter buy and sell during each of the next six months to maximize his profit trading soybeans?

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

38. Jack Potts recently won $1,000,000 in Las Vegas and is trying to determine how to invest his winnings. He has narrowed his decision down to five investments, which are summarized in the following table.

Summary of Cash Inflows and Outflows (at beginning of years)

	1	2	3	4
A	−1	0.50	0.80	
B		−1	⟷	1.25
C	−1	⟷	⟷	1.35
D			−1	1.13
E	−1	⟷	1.27	

If Jack invests $1 in investment A at the beginning of year 1, he will receive $0.50 at the beginning of year 2 and another $0.80 at the beginning of year 3. Alternatively, he can invest $1 in investment B at the beginning of year 2 and receive $1.25 at the beginning of year 4. Entries of "⟷" in the table indicate times when no cash inflows or outflows can occur. At the beginning of any year, Jack can place money in a money market account that is expected to yield 8% per year. He wants to keep at least $50,000 in the money market account at all times and doesn't want to place any more than $500,000 in any single investment. How would you advise Jack to invest his winnings if he wants to maximize the amount of money he'll have at the beginning of year 4?

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

39. Fred and Sally Merrit recently inherited a substantial amount of money from a deceased relative. They want to use part of this money to establish an account to pay for their daughter's college education. Their daughter, Lisa, will be starting college 5 years from now. The Merrits estimate that her first year college expenses will amount to $12,000 and increase $2,000 per year during each of the remaining three years of her education. The following investments are available to the Merrits:

Investment	Available	Matures	Return at Maturity
A	Every year	1 year	6%
B	1, 3, 5, 7	2 years	14%
C	1, 4	3 years	18%
D	1	7 years	65%

The Merrits want to determine an investment plan that will provide the necessary funds to cover Lisa's anticipated college expenses with the smallest initial investment.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

40. Refer to the previous question. Suppose the investments available to the Merrits have the following levels of risk associated with them.

Investment	A	B	C	D
Risk Factor	1	3	6	8

If the Merrits want the weighted average risk level of their investments to not exceed 4, how much money will they need to set aside for Lisa's education and how should they invest it?

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

41. WinterWearhouse operates a clothing store specializing in ski apparel. Given the seasonal nature of its business, often there is somewhat of an imbalance between when bills must be paid for inventory purchased and when the goods actually are sold and cash is received. Over the next six months, the company expects cash receipts and requirements for bill paying as follows:

	Month					
	1	**2**	**3**	**4**	**5**	**6**
Cash Receipts	\$100,000	\$225,000	\$275,000	\$350,000	\$475,000	\$625,000
Bills Due	\$400,000	\$500,000	\$600,000	\$300,000	\$200,000	\$100,000

The company likes to maintain a cash balance of at least \$20,000 and currently has \$100,000 cash on hand. The company can borrow money from a local bank for the following term/rate structure: 1 month at 1%, 2 months at 1.75%, 3 months at 2.49%, 4 months at 3.22%, and 5 months at 3.94%. When needed, money is borrowed at the end of a month and repaid, with interest, at the end of the month in which the obligation is due. For instance, if the company borrows \$10,000 for 2 months in month 3, it would have to pay back \$10,175 at the end of month 5.

a. Create a spreadsheet model for this problem and solve it.
b. What is the optimal solution?
c. Suppose its bank limits WinterWearhouse to borrowing no more than \$100,000 at each level in the term/rate structure. How would this restriction change the optimal solution?

42. The accounting firm of Coopers & Andersen is conducting a benchmarking survey to assess the satisfaction level of its clients versus clients served by competing accounting firms. The clients are divided into four groups:

Group 1: Large clients of Coopers & Andersen
Group 2: Small clients of Coopers & Andersen
Group 3: Large clients of other accounting firms
Group 4: Small clients of other accounting firms

A total of 4,000 companies are being surveyed either by telephone or via a two-way web-cam interview. The costs associated with surveying the different types of companies are summarized below:

	Survey Costs	
Group	**Telephone**	**Webcam**
1	\$18	\$40
2	\$14	\$35
3	\$25	\$60
4	\$20	\$45

Coopers & Andersen wants the survey to carry out the survey in the least costly way that meets the following conditions:

- At least 10% but not more than 50% of the total companies surveyed should come from each group.
- At least 50% of the companies surveyed should be clients of Coopers & Andersen.
- At least 25% of the surveys should be done via web cam.
- At least 50% of the large clients of Coopers & Anderson who are surveyed should be done via web cam.
- A maximum of 40% of those surveyed may be small companies.
- A maximum of 25% of the small companies surveyed should be done via web cam.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

43. The chief financial officer for Eagle's Beach Wear and Gift Shop is planning for the company's cash flows for the next six months. The following table summarizes the expected accounts receivables and planned payments for each of these months (in $100,000s).

	January	February	March	April	May	June
Accounts Receivable Balances Due	1.50	1.00	1.40	2.30	2.00	1.00
Planned Payments (net of discounts)	1.80	1.60	2.20	1.20	0.80	1.20

The company currently has a beginning cash balance of $400,000 and desires to maintain a balance of at least $25,000 in cash at the end of each month. To accomplish this, the company has several ways of obtaining short-term funds:

1. ***Delay Payments***. In any month, the company's suppliers permit it to delay any or all payments for one month. However, for this consideration, the company forfeits a 2% discount that normally applies when payments are made on time. (Loss of this 2% discount is, in effect, a financing cost.)
2. ***Borrow Against Accounts Receivables***. In any month, the company's bank will loan it up to 75% of the accounts receivable balances due that month. These loans must be repaid in the following month and incur an interest charge of 1.5%.
3. ***Short-Term Loan***. At the beginning of January, the company's bank will also give it a 6-month loan to be repaid in a lump sum at the end of June. Interest on this loan is 1% per month and is payable at the end of each month.

Assume the company earns 0.5% interest each month on cash held at the beginning of the month. Create a spreadsheet model that the company can use to determine the least costly cash management plan (*i.e.*, minimal net financing costs) for this 6-month period. What is the optimal solution?

44. The DotCom Corporation is implementing a pension plan for its employees. The company intends to start funding the plan with a deposit of $50,000 on January 1, 2008. It plans to invest an additional $12,000 one year later, and continue making additional investments (increasing by $2,000 per year) on January 1 of each year from 2010 through 2022. To fund these payments, the company plans to purchase a

number of bonds. Bond 1 costs $970 per unit and will pay a $65 coupon on January 1 of each year from 2009 through 2012 plus a final payment of $1065 on January 1, 2013. Bond 2 costs $980 and will pay a $73 coupon on January 1 of each year from 2009 through 2018 plus a final payment of $1073 on January 1, 2019. Bond 3 costs $1025 and will pay a $85 coupon on January 1 of each year from 2009 through 2021 plus a final payment of $1085 on January 1, 2022. The company's cash holdings earn an interest rate of 4.5%. Assume that the company wants to purchase bonds on January 1, 2008 and cannot buy them in fractional units. How much should the company invest in the various bonds and cash account to fund this plan through January 1, 2022 in the least costly way?

a. Create a spreadsheet model for this problem and solve it.
b. What is the optimal solution?

45. A natural gas trading company wants to develop an optimal trading plan for the next 10 days. The following table summarizes the estimated prices (per thousand cubic feet (cf)) at which the company can buy and sell natural gas during this time. The company may buy gas at the "Ask" price and sell gas at the "Bid" price.

Day	1	2	3	4	5	6	7	8	9	10
Bid	$3.06	$4.01	$6.03	$4.06	$4.01	$5.02	$5.10	$4.08	$3.01	$4.01
Ask	$3.22	$4.10	$6.13	$4.19	$4.05	$5.12	$5.28	$4.23	$3.15	$4.18

The company currently has 150,000 cf of gas in storage and has a maximum storage capacity of 500,000 cf. To maintain the required pressure in the gas transmission pipeline system, the company can inject no more than 200,000 cf into the storage facility each day and can extract no more than 180,000 cf per day. Assume that extractions occur in the morning and injections occur in the evening. The owner of the storage facility charges a storage fee of 5% of the market (bid) value of the average daily gas inventory. (The average daily inventory is computed as the average of each day's beginning and ending inventory.)

a. Create a spreadsheet model for this problem and solve it.
b. What is the optimal solution?
c. Assuming price forecasts for natural gas change on a daily basis, how would you suggest that the company in this problem actually use your model?

46. The Embassy Lodge hotel chain wants to compare its brand efficiency to that of its major competitors using DEA. Embassy collected the following data reported in industry trade publications. Embassy views customers' perceptions of satisfaction and value (scored from 0 to 100 where 100 is best) to be outputs produced as a function of the following inputs: price, convenience, room comfort, climate control, service, and food quality. (All inputs are expressed on scales where less is better.)

Brand	Satisfaction	Value	Price	Convenience	Room Comfort	Climate Control	Service	Food Quality
Embassy Lodge	85	82	70.00	2.3	1.8	2.7	1.5	3.3
Sheritown Inn	96	93	70.00	1.5	1.1	0.2	0.5	0.5
Hynton Hotel	78	87	75.00	2.2	2.4	2.6	2.5	3.2
Vacation Inn	87	88	75.00	1.8	1.6	1.5	1.8	2.3
Merrylot	89	94	80.00	0.5	1.4	0.4	0.9	2.6
FairPrice Inn	93	93	80.00	1.3	0.9	0.2	0.6	2.8
Nights Inn	92	91	85.00	1.4	1.3	0.6	1.4	2.1
Western Hotels	97	92	90.00	0.3	1.7	1.7	1.7	1.8

a. Compute the DEA efficiency for each brand.
b. Which brands are efficient?
c. Is Embassy Lodge efficient? If not, what input and output values should it aspire to, to become efficient?

47. Fidelity Savings & Loans (FS&L) operates several banking facilities throughout the Southeastern United States. The officers of FS&L want to analyze the efficiency of the various branch offices using DEA. The following data has been selected to represent appropriate input and output measures of each banking facility.

Branch	R.O.A.	New Loans	Satisfaction	Labor Hrs	Op. Costs
1	5.32	770	92	3.73	6.34
2	3.39	780	94	3.49	4.43
3	4.95	790	93	5.98	6.31
4	6.01	730	82	6.49	7.28
5	6.40	910	98	7.09	8.69
6	2.89	860	90	3.46	3.23
7	6.94	880	89	7.36	9.07
8	7.18	970	99	6.38	7.42
9	5.98	770	94	4.74	6.75
10	4.97	930	91	5.04	6.35

a. Identify the inputs and outputs for FS&L. Are they all measured on the appropriate scale for use with DEA?
b. Compute the DEA efficiency of each branch office.
c. Which offices are DEA efficient?
d. What input and output levels should branch 5 aspire to, to become efficient?

CASE 3.1 Putting the Link in the Supply Chain

Rick Eldridge is the new Vice President for operations at the The Golfer's Link (TGL), a company specializing in the production of quality, discount sets of golf clubs. Rick was hired primarily because of his expertise in supply chain management (SCM). SCM is the integrated planning and control of all resources in the logistics process from the acquisition of raw materials to the delivery of finished products to the end user. Because SCM seeks to optimize all activities in the supply chain including transactions between firms, Rick's first priority is ensuring that all aspects of production and distribution within TGF are operating optimally.

TGL produces three different lines of golf clubs for men, women, and junior golfers at manufacturing plants in Daytona Beach, FL, Memphis, TN, and Tempe, AZ. The plant in Tempe produces all three lines of clubs. The one in Daytona produces only the men's and women's lines, and the plant in Memphis produces only the women's and juniors' lines. Each line of clubs requires varying amounts of three raw materials that are sometimes in short supply: titanium, aluminum, and a distinctive rock maple wood that TGL uses in all of its drivers. The manufacturing process for each line of clubs at each plant is identical. Thus, the amount of each of these materials required in each set of the different lines of clubs is summarized below:

	Resources Required per Club Set (in lbs)		
	Men's	Women's	Juniors'
Titanium	2.9	2.7	2.5
Aluminum	4.5	4	5
Rock Maple	5.4	5	4.8

The estimated amount of each of these key resources available at each plant during the coming month is given as:

	Estimated Resource Availability (in lbs)		
	Daytona	**Memphis**	**Tempe**
Titanium	4500	8500	14500
Aluminum	6000	12000	19000
Rock Maple	9500	16000	18000

TGL's reputation for quality and affordability ensures that it can sell all the clubs it can make. The men's, women's, and juniors' lines generate wholesale revenues of $225, $195, and $165, respectively, regardless of where they are produced. Club sets are shipped from the production plants to distribution centers in Sacramento, CA, Denver, CO, and Pittsburgh, PA. Each month, the different distribution centers order the number of club sets in each of the three lines that they would like to receive. TGL's contract with this distributor requires filling at least 90% (but no more than 100%) of all distributor orders. Rick recently received the following distributor orders for the coming month:

	Number of Club Sets Ordered		
	Men's	**Women's**	**Juniors'**
Sacramento	700	900	900
Denver	550	1000	1500
Pittsburgh	900	1200	1100

The cost of shipping a set of clubs to each distribution point from each production facility is summarized in the following table. Note again that Daytona does not produce juniors' club sets and Memphis does not produce men's club sets.

	Shipping Costs						
	Men's		**Women's**			**Juniors'**	
To/From	**Daytona**	**Tempe**	**Daytona**	**Memphis**	**Tempe**	**Memphis**	**Tempe**
Sacramento	$51	$10	$49	$33	$9	$31	$8
Denver	$28	$43	$27	$22	$42	$21	$40
Pittsburgh	$36	$56	$34	$13	$54	$12	$52

Rick has asked you to determine an optimal production and shipping plan for the coming month.

a. Create a spreadsheet model for this problem and solve it. What is the optimal solution?
b. If Rick wanted to improve this solution, what additional resources would be needed and where would they be needed? Explain.
c. What would TGL's optimal profit be if they were not required to supply at least 90% of each distributor's order?
d. Suppose TGL's agreement included the option of paying a $10,000 penalty if they cannot supply at least 90% of each the distributor's order but instead supply at least 80% of each distributor's order. Comment on the pros and cons of TGL exercising this option.

CASE 3.2

Foreign Exchange Trading at Baldwin Enterprises

Baldwin Enterprises is a large manufacturing company with operations and sales divisions located in the United States and several other countries. The CFO of the organization, Wes Hamrick, is concerned about the amount of money Baldwin has been paying in transaction costs in the foreign exchange markets and has asked you to help optimize Baldwin's foreign exchange treasury functions.

With operations in several countries, Baldwin maintains cash assets in several different currencies: U.S. dollars (USD), the European Union's euro (EUR), Great Britain's pound (GBP), Hong Kong dollars (HKD), and Japanese yen (JPY). To meet the different cash flow requirements associated with its operations around the world, Baldwin often must move funds from one location (and currency) to another. For instance, to pay an unexpected maintenance expense at its facility in Japan, Baldwin might need to convert some of its holdings in U.S. dollars to Japanese yen.

The foreign exchange (FX) market is a network of financial institutions and brokers in which individuals, businesses, banks, and governments buy and sell the currencies of different countries. They do so to finance international trade, invest or do business abroad, or speculate on currency price changes. The FX market operates 24 hours a day and represents the largest and most liquid marketplace in the global economy. On average, the equivalent of about $1.5 trillion in different currencies is traded daily in the FX market around the world. The liquidity of the market provides businesses with access to international markets for goods and services by providing foreign currency necessary for transactions worldwide (see:http://www.ny.frb.org/fxc).

The FX market operates in much the same way as a stock or commodity market; there is a bid price and ask price for each commodity (or, in this case, currency). A bid price is the price at which the market is willing to buy a particular currency and the ask price is the price at which the market is willing to sell a currency. The ask prices are typically slightly higher than the bid prices for the same currency—representing the transaction cost or the profit earned by the organizations that keep the market liquid.

The following table summarizes the current FX rates for the currencies Baldwin currently holds. The entries in this table represent the conversion rates from the row currencies to the column currencies.

Convert/To	USD	EUR	GBP	HKD	JPY
USD	1	1.01864	0.6409	7.7985	118.55
EUR	0.9724	1	0.6295	7.6552	116.41
GBP	1.5593	1.5881	1	12.154	184.97
HKD	0.12812	0.1304	0.0821	1	15.1005
JPY	0.00843	0.00856	0.0054	0.0658	1

For example, the table indicates that one British pound (GBP) can be exchanged (or sold) for 1.5593 U.S. dollars (USD). Thus, $1.5593 is the bid price, in U.S. dollars, for one British pound. Alternatively, the table indicates one U.S. dollar (USD) can be exchanged (sold) for 0.6409 British pounds (GBP). So, it takes about 1.5603 U.S. dollars (or 1/0.6409) to buy one British pound (or the ask price, in U.S. dollars, for one British pound is roughly $1.5603).

Notice that if you took one British pound, converted it to 1.5593 U.S. dollars, and then converted those 1.5593 dollars back to British pounds, you would end up with only

0.999355 British pounds (i.e., $1 \times 1.5593 \times 0.6409 = 0.999355$). The money that you lose in this exchange is the transaction cost.

Baldwin's current portfolio of cash holdings includes 2 million USD, 5 million EUR, 1 million GBP, 3 million HKD, and 30 million JPY. This portfolio is equivalent to \$9,058,710 USD under the current exchange rates (given above). Wes has asked you to design a currency trading plan that would increase Baldwin's euro and yen holdings to 8 million EUR and 54 JPY, respectively, while maintaining the equivalent of at least \$250,000 USD in each currency. Baldwin measures transaction costs as the change in the USD equivalent value of the portfolio.

a. Create a spreadsheet model for this problem and solve it.
b. What is the optimal trading plan?
c. What is the optimal transaction cost (in equivalent USD)?
d. Suppose that another executive thinks that holding \$250,000 USD in each currency is excessive and wants to lower the amount to \$50,000 USD in each currency. Does this help to lower the transaction cost? Why or why not?
e. Suppose the exchange rate for converting USD to GBP increased from 0.6409 to 0.6414. What would happen to the optimal solution in this case?

The Wolverine Retirement Fund

CASE 3.3

Kelly Jones is a financial analyst for Wolverine Manufacturing, a company that produces engine bearings for the automotive industry. Wolverine is hammering out a new labor agreement with its unionized workforce. One of the major concerns of the labor union is the funding of Wolverine's retirement plan for their hourly employees. The union believes that the company has not been contributing enough money to this fund to cover the benefits it will need to pay to retiring employees. Because of this, the union wants the company to contribute approximately \$1.5 million dollars in additional money to this fund over the next 20 years. These extra contributions would begin with an extra payment of \$20,000 at the end of one year with annual payments increasing by 12.35% per year for the next 19 years.

The union has asked the company to set up a sinking fund to cover the extra annual payments to the retirement fund. Wolverine's Chief Financial Officer and the union's chief negotiator have agreed that AAA-rated bonds recently issued by three different companies may be used to establish this fund. The following table summarizes the provisions of these bonds.

Company	Maturity	Coupon Payment	Price	Par Value
AC&C	15 years	\$80	\$847.88	\$1,000
IBN	10 years	\$90	\$938.55	\$1,000
MicroHard	20 years	\$85	\$872.30	\$1,000

According to this table, Wolverine may buy bonds issued by AC&C for \$847.88 per bond. Each AC&C bond will pay the bondholder \$80 per year for the next 15 years, plus an extra payment of \$1,000 (the par value) in the fifteenth year. Similar interpretations apply to the information for the IBN and MicroHard bonds. A money market fund yielding 5% may be used to hold any coupon payments that are not needed to meet the company's required retirement fund payment in any given year.

Wolverine's CFO has asked Kelly to determine how much money the company would have to invest and which bonds the company should buy to meet the labor union's demands.

a. If you were Kelly, what would you tell the CFO?
b. Suppose that the union insists on including one of the following stipulations into the agreement:
   1. No more than half of the total number of bonds purchased may be purchased from a single company.
   2. At least 10% of the total number of bonds must be purchased from each of the companies.

Which stipulation should Wolverine agree to?

CASE 3.4 Saving the Manatees

"So how am I going to spend this money," thought Tom Wieboldt as he sat staring at the pictures and posters of manatees around his office. An avid environmentalist, Tom is the president of "Friends of the Manatees"—a nonprofit organization trying to help pass legislation to protect manatees.

Manatees are large, gray-brown aquatic mammals with bodies that taper to a flat, paddle-shaped tail. These gentle and slow-moving creatures grow to an average adult length of 10 feet and weigh an average of 1,000 pounds. Manatees are found in shallow, slow-moving rivers, estuaries, saltwater bays, canals, and coastal areas. In the United States, manatees are concentrated in Florida in the winter, but can be found in summer months as far west as Alabama and as far north as Virginia and the Carolinas. They have no natural enemies, but loss of habitat is the most serious threat facing manatees today. Most human-related manatee deaths occur from collisions with motor boats.

Tom's organization has been supporting a bill before the Florida legislature to restrict the use of motor boats in areas known to be inhabited by manatees. This bill is scheduled to come up for a vote in the legislature. Tom recently received a phone call from a national environmental protection organization indicating that it will donate $300,000 to Friends of the Manatees to help increase public awareness about the plight of the manatees, and to encourage voters to urge their representatives in the state legislature to vote for this bill. Tom intends to use this money to purchase various types of advertising media to "get the message out" during the four weeks immediately preceding the vote.

Tom is considering several different advertising alternatives: newspapers, TV, radio, billboards, and magazines. A marketing consultant provided Tom with the following data on the costs and effectiveness of the various types of media being considered.

Advertising Medium	Unit Cost	Unit Impact Rating
Half-page, Daily paper	$800	55
Full-page, Daily paper	$1,400	75
Half-page, Sunday paper	$1,200	65
Full-page, Sunday paper	$1,800	80
Daytime TV spot	$2,500	85
Evening TV spot	$3,500	100
Highway Billboards	$750	35
15-second Radio spot	$150	45
30-second Radio spot	$300	55
Half-page, magazine	$500	50
Full-page, magazine	$900	60

According to the marketing consultant, the most effective type of advertising for this type of problem would be short TV ads during the evening prime-time hours.

Thus, this type of advertising was given a "unit impact rating" of 100. The other types of advertising were then given unit impact ratings that reflect their expected effectiveness relative to an evening TV ad. For instance, a half-page magazine ad is expected to provide half the effectiveness of an evening TV ad and is therefore given an impact rating of 50.

Tom wants to allocate the $300,000 to these different advertising alternatives in a way that will maximize the impact achieved. However, he realizes that it is important to spread his message via several different advertising channels, as not everyone listens to the radio and not everyone watches TV in the evenings.

The two most widely read newspapers in the state of Florida are the *Orlando Sentinel* and the *Miami Herald*. During the four weeks before the vote, Tom wants to have half-page ads in the daily (Monday-Saturday) versions of each of these papers at least three times per week. He also wants to have one full-page ad in the daily version of each paper the week before the vote, and he is willing to run more full-page ads if this would be helpful. He also wants to run full-page ads in the Sunday editions of each paper the Sunday before the vote. Tom never wants to run a full-page and half-page ad in a paper on the same day. So the maximum number of full and half-page ads that can be run in the daily papers should be 48 (i.e., 4 weeks × 6 days per week × 2 papers = 48). Similarly, the maximum number of full and half-page ads that can be run in the Sunday papers is eight.

Tom wants to have at least one and no more than three daytime TV ads every day during the four-week period. He also wants to have at least one ad on TV every night but no more than two per night.

There are 10 billboard locations throughout the state that are available for use during the four weeks before the vote. Tom definitely wants to have at least one billboard in each of the cities of Orlando, Tampa, and Miami.

Tom believes that the ability to show pictures of the cute, pudgy, lovable manatees in the print media offers a distinct advantage over radio ads. However, the radio ads are relatively inexpensive and might reach some people that the other ads will not reach. Thus, Tom wants to have at least two 15-second and at least two 30-second ads on the radio each day. However, he wants to limit the number of radio ads to five 15-second ads and five 30-second ads per day.

There are three different weekly magazines in which Tom can run ads. Tom wants to run full-page ads in each of the magazines at some point during the four-week period. However, he never wants to run full- and half-page ads in the same magazine in a given week. Thus, the total number of full- and half-page magazine ads selected should not exceed 12 (i.e., 4 weeks × 3 magazines × 1 ad per magazine per week = 12 ads).

Although Tom has some ideas about the minimum and maximum number of ads to run in the various types of media, he's not sure how much money this will take. And if he can afford to meet all the minimums, he's really confused about the best way to spend the remaining funds. So again Tom asks himself, "How am I going to spend this money?"

a. Create a spreadsheet model for this problem and solve it. What is the optimal solution?
b. Of the constraints Tom placed on this problem, which are "binding" or preventing the objective function from being improved further?
c. Suppose Tom was willing to increase the allowable number of evening TV ads. How much would this improve the solution?
d. Suppose Tom was willing to double the allowable number of radio ads aired each day. How much would this improve the solution?